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HYPERFINE STRUCTURES OF LANTHANUM LINES*

By William F. Meggers and Keivin Burns

I. INTRODUCTION

In the course of an analysis of the ordinary multiplet structures in the La I and La II spectra, recently published by one of us,[1] Dr. King kindly placed his data on the temperature classification of lanthanum lines at our disposal. In King's tables many lines were marked *d* or *t* denoting respectively unresolved doublets or triplets, and he remarked that approximately 18 per cent of the whole number of lanthanum lines listed appeared to be double. When these fine-structure notes were written down on the ordinary multiplet structures, they immediately suggested that certain of the identified energy levels possessed hyperfine structure. Since indications of more or less similar hyperfine structure were present in both spectra, La I and La II, it appeared that further investigation of these might be expected to throw some light on the nature and cause of these phenomena. A detailed study was accordingly begun, and a preliminary examination of several hundred lines has been made. The complete investigation of this problem is a larger and more difficult undertaking than was at first anticipated—it will require several years of careful observing—but the results of our preliminary work are thought to be of sufficient interest to be presented at this time.

* Published by permission of the Director of the Bureau of Standards of the Department of Commerce.

[1] Meggers, Journ. Wash. Acad. Sci., *17*, p. 25; 1927. La I spectrum.
Meggers, J.O.S.A. & R.S.I., *14*, p. 191; 1927. La II spectrum.

II. APPARATUS AND METHOD

In this investigation a vacuum arc like that described by Curtis[2] was used as a source, a cored graphite rod filled with lanthanum chloride or oxide[3] mixed with powdered graphite serving as a lower electrode, and a copper rod as the upper electrode. The arc was operated with about 10 amperes, 220 volt direct current, in a residual gas pressure of less than 5 cm equivalent mercury pressure. This type of source made the spectral lines and components of complex lines very sharp or narrow, and eliminated any tendency of the lines to reverse themselves.

The spectrographs employed were either a diffraction grating alone, or a grating combined with a Fabry and Perot interferometer of 5, 6, 7.5 or 15 mm plate separations. In the former case a practical resolving power of about 150,000 was attained in the second order spectrum. For lines of comparable intensity, the 6 mm interferometer crossed with a lower dispersion grating resolved about 0.2 of an interference order, that is, its practical resolving power was about 100,000 for wave-length 6000A. Under the same conditions the resolving power of the 15 mm interferometer should be 250,000 but in practice it is somewhat less than this, probably on account of astigmatism in the grating images. Thus far, only single lines have been measured with the 15 mm interferometer. The total wave-length range in which interference spectrograms have been studied extends from 3800 to 6800A, while the grating observations have been restricted by the instrument mounting to the interval 3800–4500A. The wave-lengths of about 400 lines have been measured relative to neon standards, and many more were examined, but found to be unmeasurable because of unresolved structure. Roughly 30 per cent of the lines in each spectrum, La I and La II, are complex in structure, the components thus far measured being usually spaced at distances of a few hundredths of an Angstrom. The study of hyperfine structure in any case is attended with many difficulties, and in a spectrum containing many hundreds of lines with a very large range of intensities it is especially tedious to find the optimum conditions of exposure and order of interference for the full resolution of each complex line. In this investigation the separations of the hyperfine levels are so small that the identity of the sublevels could scarcely be determined from separations alone, these being too near the error of observation. All of these sublevels have

[2] Curtis, J.O.S.A. & R.S.I., *8*, p. 697; 1924.

[3] We are indebted to Professor B. S. Hopkins of the University of Illinois for his generous gift of lanthanum salt used in this investigation.

been determined independently from combinations with other levels which are in other combinations shown to be quite sharp. It has been necessary, therefore, to determine with considerable precision, the wave-lengths of a large number of sharp lines in order to study the fine structure of those lines which display this characteristic. And the further analysis of hyperfine structure must depend not only on better resolution of the complex lines, but also on a further refinement of the wave-lengths of those lines showing no structure. A complete report of these measurements will be published at a later date.

III. RESULTS

It has already been shown that the spectrum of neutral lanthanum atoms is described by energy states with even multiplicities, doublet and quartet spectral terms having been recognized from Zeeman effects. Furthermore, it was established that the term of lowest energy—the normal state of the La atom—was represented by a doublet D, and the first metastable state by a quartet $\overline{F}$ term. The spectrum of ionized lanthanum was found to be characterized by odd multiplicities, viz., singlets and triplets, and the zero energy state is represented by a triplet $\overline{F}$ term. Metastable states have been established as follows, in the order of increasing energy, a^1D, ^3D, $^3\overline{P}$, ^1S, ^1G, b^1D.

Many higher levels have been found in both spectra, but they are of no interest here since the above described observations indicate that the full responsibility for hyperfine structure of lanthanum lines rests upon the low metastable states. Thus, in both the La I and the La II spectrum, it appears that the lowest levels (normal states) ^2D and $^3\overline{F}$ respectively, are remarkably sharp, but the first metastable states, $^4\overline{F}$ and ^1D, respectively, are clearly complex. Similar complexity has been established also for the ^3D metastable state in the La II spectrum, but the remaining metastable terms are either sharp or nearly so. Thus, the low $^3P_0$, $^3P_1$, b^1D_2 and $^1G_4$ levels are probably single; the $^3\overline{P}_2$ and $^1S_0$ levels are suspected of being very close triples but neither of them has been satisfactorily separated with the resolving powers so far employed.

Returning to the metastable states exhibiting undoubted hyperfine structure it is important to note that the various levels of a polyfold term have very distinctive fine structure features. Thus, in the La I spectrum, $^4\overline{F}_2$ has been measured twice as a quartet, $^4\overline{F}_3$ is either single or has its components very closely packed, $^4\overline{F}_4$ has been observed double in five instances, and $^4\overline{F}_5$ has not yet been fully resolved, having been

measured triple in three instances but suspected of being quadruple each time. Similarly in the La II spectrum, the three levels of the ^3D metastable term which are involved in the production of most of the strong lines, have quite different fine structure characteristics; $^3D_1$ has at least three fine structure levels, $^3D_2$ is quite sharp, the average deviation from the mean value of the level being only one part in two millions for nine different lines, and $^3D_3$ is probably at least quintuple but only one combination, that with a^3F_4, has been reasonably well resolved.

TABLE 1. *Terms in the La I and La II Spectra.*

Spectrum	Term symbol	Term value	Notes	Electron configuration
La I	a^3D_2	0.000		$5d$, $6s^2$
	a^3D_3	1053.158		
	$a^4\overline{F}_2$	2667.96	Interval ratios 9:7:5	
		8.15		$5d^2$, $6s$
		8.30		
		8.42		
	$a^4\overline{F}_3$	3010.00	Close double ?	
	$a^4\overline{F}_4$	3494.455		
		4.633		
	$a^4\overline{F}_5$	4121.6	quartet ?, not resolved	
La II	$a^3\overline{F}_2$	0.000		$5d^2$
	$a^3\overline{F}_3$	1016.074		
	$a^3\overline{F}_4$	1970.687		
	a^1D_2	1394.298		$5d$, $6s$
		4.476		
		4.622		
	a^3D_1	1894.967		$5d$, $6s$
		5.168		
		5.342		
	a^3D_2	2591.600		
	a^3D_3	3250.00		
		0.15		
		0.31		
		0.49		
		0.71		
	$a^3\overline{P}_0$	5249.67		$5d^2$
	$a^3\overline{P}_1$	5718.11		
	$a^3\overline{P}_2$	6227.34	Uncertain	
		7.52	Uncertain	
	a^1S_0	7394.48	Uncertain	$5d^2$
		4.59	Uncertain	
	a^1G_4	7473.29		$5d^2$
	b^1D_2	10094.90		$5d^2$

The results are summarized in Table 1 which presents the relative values of the term levels and hyperfine sublevels as derived from the preliminary measurements described above. It is emphasized that these results are subject to correction and will be extended by further measurements.

Disregarding for the present, the hyperfine sublevels which are in doubt, we see an apparent relation between fine structure and electron configuration. The terms in which the j (inner quantum number) levels are split up into f (fine quantum number) sublevels are those which result from the configuration containing *one* s electron. Thus, in the La I spectrum, the term a^2D produced by one d and *two* s electrons has no hyperfine structure, but the metastable $a^4\overline{F}$ term resulting from two d and *one* s electron is decidedly complex. In the La II spectrum the terms a^1D and a^3D arising from the configuration of one d and *one* s electron are very complex while the remaining terms, all assigned to two d electrons are sharp or nearly so. It will be of interest to investigate the fine structures of successive terms in a sequence which converges in the limit to sharp or complex terms of the next spectrum. For example, ^2D of La I might be expected to show fine structure in higher terms since this series converges to ^3D of La II which is complex, while the observed structure in $^4\overline{F}$ of La I will probably contract in terms approaching the sharp ^3F of La II as a limit.

The same relation between electron configuration and hyperfine structure appears to exist in several other spectra.[4] Thus, in Hg the normal state ^1S $(6s^2)$ is single while the terms ^1P, ^3P $(6s, 6p)$ are very complex. In the Cd spectrum, the term ^3P has complex levels like Hg, but the singlet terms appear to differ somewhat from their Hg analogues in that ^1P $(5s, 5p)$ is single while ^1S $(5s^2)$, according to MacNair,[5] is double. The only fine structures which heretofore have been well observed in the spectrum of an ionized atom are those given by Paschen for Al II. Here again the terms ^1F, ^3F $(3s, 4f)$ are found to be complex.

It does not, however, appear possible to ascribe all hyperfine structure to the combined action of a single s electron and one or more electrons of different types because in addition to the ^1P term of Cd just mentioned, other exceptions occur in the arc spectra of Cu and Mn. The normal state ^2S $(3d^{10}, 4s)$ of Cu is represented by complex levels, but so

[4] Ruark and Chenault, Phil. Mag., *1*, p. 937; 1925. Fine structures of spectrum lines of Cu, Cd, Hg, Al^+, In, Tl, Pb, Bi, Mn, Cl, Br, I.

Ruark, Phil. Mag., *1*, p. 977; 1925. Fine structure and Zeeman effect of complex Hg lines.

[5] MacNair, Phil. Mag., *2*, p. 613; 1926.

also is the metastable state ^2D ($3d^9$, $4s^2$). In Mn, the normal state ^6S ($3d^5$, $4s^2$) is complex, the metastable state ^6D ($3d^6$, $4s$) has simple levels, and two other terms, ^6P ($3d^5$, $4s$, $4p$) and $^6\overline{D}$ ($3d^6$, $4p$) have been described as complex levels.

Professor H. N. Russell has suggested in a letter to one of the authors that instead of electron responsibility for hyperfine structure, a spinning proton may be the cause of these phenomena. The latter should give separations of the order of magnitude of 1/2000 of the ordinary multiplet separations, and this seems to be roughly the case. The fact that the intervals between successive fine levels often show a progression like that given by Landé's interval rule suggests that we are dealing with a new kind of quantized inclination of angular momentum vectors in which the atom "Rumpf" probably plays the leading part. Such an effect should vary as $Z_i Z_a^2$ where Z_i is the charge effective on the inner part of the orbit and Z_a on the outer. This would account for the relatively wide hyperfine structures in spectra of the heavier atoms, their shrinkage for the lighter elements, and would also give fine separations in wave-length of about the same magnitude in the spectra of neutral atoms and of successive stages of ionization. It is conceivable, furthermore, that fine structure would be more prominent in terms arising from configurations with a single unbalanced s electron for when the spin is in the nucleus the S doublet might have a finite width, much greater than any other, while a pair of s electrons which gives no resulting angular momentum should not take part in any space quantization effects.

This, however, does not account for the above mentioned anomalies, nor for the fact that certain levels of a polyfold term should appear single while others are very complex. The hyperfine-structure data available at present are perhaps not extensive enough to permit a satisfactory theoretical interpretation of the phenomena.

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MARCH, 1927.

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ON THE SPECTRUM OF Si^+II

By P. K. Kichlu

It was shown by Prof. A. Fowler[1] that the spectrum of Si^+ contains a doublet series of S, P, D and F terms, and a number of other terms the nature of which could not be determined. He was further able to trace its similarity with the spectrum of aluminium, the element which precedes it in the periodic table. In a letter to Nature of October 31, 1925, Prof. M. N. Saha identified a group of lines in the yellow region as originating from a quartet $P\ \overline{P}$ combination. These and other terms obtained by Professors Fowler and Saha receive a ready explanation on the Hund[2] theory of complicated spectra.

The electronic configuration in an atom of singly ionized silicon may conveniently be represented according to the scheme proposed by Prof. Saha.[3] This manner of representation brings out the energies of electrons in different states very clearly. The electrons have energies of the same, though progressively decreasing, order of magnitude, as we go down to various states in the same vertical column.

Structure diagram of Si^+ (13).

K_1						
2						
	L_1	L_2				
	2	6				
		M_1	M_2	M_3		
		2	(1)	(1)		
			N_1	N_2	N_3	N_4
			(1)	(1)	(1)	(1)
				O_1	O_2	O_3
				(1)	(1)	(1)

N. B. In this scheme, the higher L_2, M_2, M_3 · · ·levels have not been split into sublevels like L_{21} L_{22}, M_{21}, M_{22}. On this point, see a paper by M. N. Saha and B. B. Ray in the Physikalische Zeitschrift.

The normal levels of Si^+ arise in the following ways: (a) $2M_1M_2$ (two electrons in M_1, one in M_2), give the fundamental 1^2P-terms of once ionized silicon. Higher Rydberg sequence of these terms may be obtained by putting the running electron in N_2, O_2, P_2 · · · respectively; (b) the next higher set of levels is obtained by the combination

[1] A. Fowler, Phil. trans., *225*, p. 20.

[2] F. Hund, ZS. f. Physik., *33*, p. 345; *34*, p. 296.

[3] Vide P. K. Kichlu and M. N. Saha. On the Spectra of Metals of Group II (communicated to the Philosophical Magazine).

$2M_1N_1$ giving a $^2S_1$ term. The Rydberg sequence is obtained with the combinations $2M_1O_1$, $2M_1P_1 \cdots$ etc. (c) $2M_1M_2$ give rise to ^2D terms. M_2 being a slightly higher position than N_2, the values of the first members of the ^2D-series should be greater than the second members of P-series. $2M_1$ combined with N_2, O_2, etc., provide a Rydberg sequence of the ^2D terms. (d) $2M_1N_4$ combination gives us 1^2F terms. Our diagram illustrates that the values of these terms must lie between $2M_1N_3$ and $2M_1O_3$ combinations. The higher members of this series are obtained by placing the running electron in O_4, $P_4 \cdots$.

All these terms are regular terms, and have been obtained exactly in the form indicated by the theory. The detailed identification is shown in Table 1.

TABLE 1.

State	Term designation	Term values	
		Al-values	Si^+-values/4
$2M_1\,M_2$	$1^2P_{1,2}$	48280.9	32954
		48168.9	32883
$2M_1\,N_2$	$2^2P_{1,2}$	15331.7	12658
		15316.5	12643
$2M_1\,O_2$	$3^2P_{1,2}$	8009.2	6990
		8003.2	6984
$2M_1\,P_2$	$4^2P_{1,2}$	4946.0	4442
		4943.2	4440
$2M_1\,N_1$	1^2S_1	22933	16581
$2M_1\,O_1$	2^2S_1	10591	8463
$2M_1\,P_1$	3^2S_1	6137	5160
$2M_1\,Q_1$	4^2S_1	4007	3477
$2M_1\,M_2$	$1^2D_{2,3}$	15845	13121
			13116
$2M_1\,N_2$	$2^2D_{2,3}$	9352	7700
$2M_1\,O_2$	$3^2D_{2,3}$	6047	4857
$2M_1\,P_2$	$4^2D_{2,3}$	4114	3325
$2M_1\,N_4$	$1^2F_{3,4}$	6962	7066
$2M_1\,O_4$	$2^2F_{3,4}$	4451	4515
$2M_1\,P_4$	$3^2F_{3,4}$		3127

But along with these regular terms there are a number of other terms obtained by Professors Fowler and Saha which may be interpreted in the manner given below:

(e) $M_1 2M_2$ (one electron in M_1 and two in M_2) give us the terms

$$^4\overline{P}_{1,2,3} \qquad ^2D_{2,3} \qquad ^2S_1$$
$$^2\overline{P}_{1,2}$$

We can arrive at this step by the transition

$$2M_1M_2 \rightarrow M_1 2M_2$$
$$M_1 \rightarrow M_2$$

which is allowed. Hence the P-terms arising from the present state will be capable of combining with the P-states from the combination $2M_1M_2$. The present P-terms will, therefore, be dashed.

(f) Another possible combination is given by $M_1M_2N_1$ giving rise to a^2P, ^4P, b^2P terms.

Terms arising from (e) and (f) have been thus identified.

Fowler has given a term 'x' with doublet 'd' characteristics, which combines with regular ^2P and ^2F terms. We can identify this with $^2D_{2,3}$ of combination (e)

$$^2D_{2,3} = (x_2, x_1) = 76514.1,\ 76498.2$$

Fowler has also provisionally given a 'π' term which can be identified with $^2\overline{P}_{1,2}$ of (e).

$$^2\overline{P}_{1,2} = \Pi_{2,1}' = 48042,\ 47841$$

We have now to look for $^4\overline{P}_{1,2,3}$ and $^2S_1$. There are two lines

$$\begin{matrix} 5669.59\ (4) = 17633.1 \\ 5688.86\ (2) = 17573.4 \end{matrix} \quad 59.7$$

which may be identified as $^2S_1 - 2^2P_{1,2}$. This gives

$$^2S_1 = 68205.4$$

The greatest difficulty has been experienced in the identification of $^4P_{1,2,3}$. Prof. Saha[4] has identified a quartet $P\ \overline{P}$ multiplet with separations 116.0 and 199.9 for one set and 62.1 and 134.7 for the other. One of the sets must be undashed and the other must be dashed, but as no intercombination lines have yet been obtained it is difficult to trace their origin unambiguously.

Values of the terms arising from $M_1 2M_2$-state. If we compare the states

$M_1M_2M_2$	$M_1M_2N_2$	$M_1M_2O_2$ ...
$^4\overline{P}_{1,2,3}, ^2\overline{P}_{1,2}$	$2^4\overline{P}, 2^2\overline{P}$	$3^4\overline{P}, 3^2\overline{P}$
$2M_1M_2$	$2M_1N_2$	$2M_1O_2$ ...
$1^2P_{1,2}$	2^2P	3^2P

[4] M. N. Saha, Nature, loc. cit.

we find that taking any two higher terms, e.g.

$$M_1M_2O_2 \rightarrow 2M_1O_2$$
$$= (M_1M_2 - 2M_1)O_2$$
$$= {}^1P\,{}^3P \text{ of } Si^{++} - {}^1S \text{ of } Si^{++}$$
$$= -82857 \text{ for singlets}$$

and $$= -39330 \text{ for triplets}$$

Hence

$$n\begin{cases} {}^4\overline{P}_{1,2,3} \\ {}^2D_{2,3}, {}^2S \end{cases} = n^2P_{1,2} - 39330$$

$$n^2\overline{P}_{1,2} = n^2P_{1,2} - 82857$$

When $n=1$, the relation may be quite wide of the mark, though it will hold with a fair degree of accuracy for higher terms.

$$1^2\overline{P}_{1,2} = 131818 - 82857$$
$$= 48961$$

Fowler's value is 48042, 47841

$$1^4P_{1,2,3} = 131818 - 39330$$
$$= 92488$$

Fowler's value of ^2D which will be less than $^4\overline{P}_{1,2,3}$ is 76514. Thus as regards the values of the terms, they are of the right order.

$$2^4\overline{P} = 50632 - 39330$$
$$= 11302$$

The values of (f) *combinations.* Compare the sets of terms arising from the pairs of combinations:

$M_1N_1N_2$	$M_1N_1O_2$	$M_1N_1P_2$ ···
$1^4P_{1,2,3}, {}^2P_{1,2}$	$2^4P, 2^2P$	$3^4P, 3^2P$
$2M_1N_2$	$2M_1O_2$	$2M_1P_2$
$1^2P_{1\,2}$	2^2P	3^2P

Now

$$(-M_1N_1P_2 - 2M_1P_2) = (M_1N_1 - 2M_1)P_2$$
$$= (2^1S_0 - 1^1S_0) \text{ of } Si^{++} \text{ for doublets}$$
$$= (2^3S_0 - 1^1S_0) \text{ of } Si^{++} \text{ for quartets}$$

and

$$= 11660 - 256472$$
$$= -139812 \text{ for quartets}$$
$$\therefore n^4P = n^2P - 139812$$

Hence

$$1^4P = 131818 - 139812$$
$$= 7994$$

So that $^4\overline{P}$-terms of (e) have rather large values, when compared to those from the combination (f). This shows that the lines $^4\overline{P} - {}^4P$ will be in the short wave length region.

The lines $^2P - {}^4\overline{P}$ are expected at about λ2500, but though there are several lines in this region not yet accounted for, their origin is still uncertain. Fowler has assigned, apparently without sufficient reasons some to Si^{++} and neglects some as being due to impurities. The available data[5] are themselves conflicting. The region must therefore be reinvestigated, by either the electron bombardment method in which the potential should be kept between 16 to 20 volts, or the vacuum arc method. It may be noted that these lines being intercombination lines occur only under very special conditions when the stimulus is restricted within narrow limits. In this connection it may be mentioned that no intercombination lines have yet been obtained between the quartet and doublet terms of O^+ and N.

The writer takes this opportunity of expressing his best thanks to Professor M. N. Saha for his kindly interest and generous help in the present investigation.

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[5] Sawyer and Paton: Astrophysical Journal, *57*, p. 279.

Modern astronomy: its rise and progress. By Hector Macpherson. 196 pages. Oxford University Press (American Branch, 35 W. 32d St., New York). $2.00.

A presentation in book form of a series of lectures delivered during 1925–26 at the Aberdeen United Free Church College. Following a historical introduction come chapters on the Moon; the Sun; the Planets; Asteroids, Comets and Meteors; the Stars; New Stars and Nebulae; Cosmology and Cosmogony. The treatment of the several topics is partly historical, but, so far as space limits permit, the reader is brought up-to-date. The result is a fascinating account of a fascinating subject—a book which will appeal to both scientist and layman.

F. K. RICHTMYER

THE MINIMA OF REGULAR REFLECTION

By H. S. Uhler

ABSTRACT

In the first part of the paper typical quotations are given to show how inadequately the theoretical side of the problem has been treated by earlier writers. The main body of the paper is devoted to the derivation of new, unexpectedly simple functions of the index of refraction which give directly the minimum value of the reflecting power and the associated values of the angles of incidence and refraction when the index is either less than $2-3^{1/2}$ or greater than $2+3^{1/2}$. This is followed by paragraphs which present, without detailed proofs, new formulae for the index of refraction and the angle of incidence in terms of the minimum value of the reflecting power, and for the index and reflecting power with the angle of incidence as independent variable. The closing paragraphs make the account complete by dealing with the behavior of the minimum of reflecting power corresponding to normal incidence, that is, to values of the index of refraction falling within the interval $2-3^{1/2}$ to $2+3^{1/2}$.

When "natural light" is regularly reflected and refracted at the surface between two transparent media it is often stated or implied in the text books on optics that the ratio of the intensity of the reflected light to the intensity of the incident light increases continuously as the angle of incidence changes from zero to ninety degrees. In the majority of cases arising in practice the preceding conclusion is true but circumstances can arise in which it is not valid. More precisely the least value of the reflecting power occurs at normal incidence when, and only when, the index of refraction n satisfies the conditions $2-3^{1/2} \leqq n \leqq 2+3^{1/2}$, or approximately $0.268 \leqq n \leqq 3.732$. When $0 \leqq n < 2-3^{1/2}$ and $2+3^{1/2} < n \leqq \infty$ there always exists a minimum of reflecting power for a finite value of the angle of incidence.

As far as the writer can find, the theory of minimum reflection has not been treated adequately and reliably. Preston[1] discriminates against internal reflection by the sentence: "The roots of the equation $\mu^2-4\mu+1=0$ are $2+3^{1/2}$ and $2-3^{1/2}$, of which the latter may be discarded (being less than unity)."

Mascart[2] says: "Setting aside normal incidence, there remains simply $\cos^3(i-i')=\cos(i+i')$. This equation is possible only when, for very small incidences, $\cos^3(i-i') < \cos(i+i')$, a condition which is equivalent to $(n-2)^2>3$, that is to $n>2+3^{1/2}=3.732$, $n<2-3^{1/2}=0.268$. The second solution is not admissible and materials are not known which have an index of refraction as high as the first. Accordingly, the coefficient of reflection is always a minimum at normal

[1] T. Preston, The Theory of Light, Art. 208; 1890, and Art. 217; 1912.

[2] M. E. Mascart, Traité d'Optique, 2, p. 417; 1891.

incidence and it presents a maximum equal to unity for grazing incidence."

To justify the last paragraph it is necessary to restrict the definition of "light" to the region of the optical spectrum and to ignore the physical significance of the fact that $2-3^{1/2}$ is the reciprocal of $2+3^{1/2}$.

A broader view of the problem is taken by Felix Jentzsch-Gräfe in a paper[3] entitled: "Minimalstellen der regulären Reflexion." On page 364 he says:

$$\text{"}\cos^3(\alpha-\beta)=\cos(\alpha+\beta)\,. \qquad (1)$$

Hence the connection between $\alpha_{\min}$ and n can be determined by taking into account the law of refraction $\sin\alpha=n\sin\beta$. If β be eliminated from equation (1) the resulting equation will be of the 24th degree in $\sin\alpha$ and of the 16th degree in n. Although only even powers occur nevertheless one gladly avoids the evaluation of an equation of the 12th degree and prefers to calculate a few values numerically as has been done for the following table I."

As a matter of fact, the writer will show that the elimination referred to by Jentzsch-Gräfe, when performed rigorously, leads to a rational algebraic equation of degree *two* in $\sin^2 i(\equiv y)$ and of degree *three* in $n^2(\equiv x)$, the degree of the equation in x and y (collectively) being only *four*. The exact formula for $\sin i_{\min}$ derived below is as simple as the approximate one given by Jentzsch-Gräfe which holds only for the limiting case $\sin\beta\doteq\beta$. An extraordinarily simple, closed, and exact formula for the minimum of reflecting power, as an explicit function of the index of refraction will also be derived. This formula too is new and it is far simpler than the corresponding one given by Jentzsch-Gräfe for the limiting case $\sin\beta\doteq\beta$.

The last paper on the subject seems to be the one by Karl Pfannenberg[4] with identically the same title as that used by Jentzsch-Gräfe. The research was wholly experimental and nothing constructive was added to the theory. Electromagnetic waves of length 25 cms were reflected from water and from aqueous solutions of sodium chloride. Within the limits of experimental error the predictions of Jentzsch-Gräfe were substantiated.

Attention will now be turned to the derivation of rigorous formulas which bring out concisely the variations of the reflecting power and of the associated parameters.

[3] Verh. d. Deutsch, Phys. Gesell., *21*, p. 361; 1919.

[4] ZS. für Physik, *37*, Heft 10/11, p. 758; 1926.

Let $i \equiv$ angle of incidence, $r \equiv$ angle of refraction, $n \equiv$ index of refraction of the medium containing r relative to the medium in which i lies, and $R \equiv$ reflecting power for unpolarized electromagnetic waves. Then, assuming the validity of Snell's law and Fresnel's formulae,

$$\sin i = n \sin r \tag{1}$$

$$R = \frac{1}{2}\sigma^2 + \frac{1}{2}\tau^2, \tag{2}$$

where

$$\sigma = -\frac{\sin (i-r)}{\sin (i+r)}, \tag{3}$$

and

$$\tau = \frac{\tan (i-r)}{\tan (i+r)}. \tag{4}$$

Consequently

$$\frac{dR}{di} = \frac{2 \tan r \sin^2 (i-r)}{\sin^2 (i+r)} \left[1 - \frac{\cos (i+r)}{\cos^3 (i-r)} \right] \tag{5}$$

Since $i = r = 0$ is not the solution now sought, the required condition for a stationary value of R is

$$\cos^3 (i-r) = \cos (i+r) \tag{6}$$

Incidentally, since r cannot be obtuse the expression preceding the first bracket in equation (5) cannot be negative so that the sign of dR/di is controlled entirely by the function between the brackets.

Since the graph of R as a function of i shows that the stationary value of R is a true minimum it would be a waste of time and printing space to demonstrate this fact formally by investigating the sign of d^2R/di^2 when $dR/di = 0$.

Substitution from (3) and (4) in (2) gives

$$R = \frac{1}{2} \left[1 + \frac{\cos^2 (i+r)}{\cos^2 (i-r)} \right] \frac{\sin^2 (i-r)}{\sin^2 (i+r)} \tag{7}$$

The expression between the brackets may be written

$$1 + \frac{\cos (i+r)}{\cos^3 (i-r)} \cos (i+r) \cos (i-r)$$

which reduces to $1 + \cos(i+r)\cos(i-r)$, that is, to $\cos^2 i + \cos^2 r$ when condition (6) is fulfilled, hence, by (7)

$$R_{\min} = \frac{(\cos^2 i + \cos^2 r) \sin^2 (i-r)}{2 \sin^2 (i+r)} \tag{8}$$

Formula (8) was published by Jentzsch-Gräfe but he did not pursue the exact analysis farther since the next step in his work involved the condition $\sin r \doteq r$. With insignificant exceptions, all results obtained in the following paragraphs are believed by the present writer to be new.

Let

$$\left.\begin{aligned} p &\equiv \sin^2 i \sin^2 r \\ s &\equiv \sin^2 i + \sin^2 r \end{aligned}\right\} \tag{9}$$

Expanding equation (6) as

$$(\cos i \cos r + \sin i \sin r)^3 = \cos i \cos r - \sin i \sin r,$$

replacing $\cos^2 i$ and $\cos^2 r$ by their respective equals $1-\sin^2 i$ and $1-\sin^2 r$, and introducing abbreviations (9) it will be found at once that

$$(s-4p)\cos i \cos r = (4p-3s+4)\sin i \sin r \tag{10}$$

Squaring equation (10) leads directly to

$$f(p,s) \equiv s^3 - s^2 - 16sp + 16p^2 + 16p = 0 \tag{11}$$

Writing $x \equiv n^2$ and $y \equiv \sin^2 i$ it follows immediately from (1) and (9) that $p = y^2/x$ and $s = y(x+1)/x$. Substituting in equation (11) and then multiplying by x^3/y^2, (since $y=0$ is not the general solution), the next equation will be obtained at once, it is

$$x^3 y - x^3 - 13x^2 y + 16xy^2 + 14x^2 - 13xy - x + y = 0 \tag{12}$$

Equation (12) is certainly not of degree 8 in x and 12 in y as stated by Jentzsch-Gräfe (*loc. cit.*).

The ground has now been prepared for obtaining the final formulae without difficulty.

$$\begin{aligned} \sin^2 (i \pm r) &= (\sin i \cos r \pm \cos i \sin r)^2 \\ &= s - 2p \pm 2 \sin i \sin r \cos i \cos r \\ &= s - 2p \pm 2p(4p-3s+4)/(s-4p), \end{aligned} \tag{13}$$

the last form (13) being obtained by substituting for $\cos i \cos r$ its value as given by equation (10).

Taking the upper sign in (13) and expanding gives

$$\sin^2 (i+r) = [(2-s)(s^2-4p) + f(p,s)]/(s-4p)$$

or, since by (11) $f(p,s) = 0$,

$$\sin^2 (i+r) = (2-s)(s^2-4p)/(s-4p). \tag{14}$$

Taking the lower sign in (13) and developing leads directly to

$$\sin^2(i-r)=(s^2-8p)/(s-4p). \tag{15}$$

Substituting from (14) and (15) in (8), and observing that $\cos^2 i+\cos^2 r=2-s$, it follows at once that

$$R_{\min}=\frac{1}{2}-\frac{2p}{s^2-4p} \tag{16}$$

Referring to identities (9) it is evident that

$$2p/(s^2-4p)=2\sin^2 i\sin^2 r/(\sin^2 i-\sin^2 r)^2$$

which is reduced by (1) to $2/\delta^2$ where

$$\delta\equiv n-\frac{1}{n}. \tag{17}$$

Finally

$$R_{\min}=\frac{1}{2}-\frac{2}{\delta^2} \tag{18}$$

The new and exact formula (18) is remarkably simple and it lends itself most readily to numerical calculations when an adequate table of reciprocals is at hand. More significant, however, is the fact that δ^2 is not affected by substituting $1/n$ for n, consequently the value of $R_{\min}$ is the same when the radiation is reflected internally (partially, not totally) as when it is reflected externally at the interface of a given pair of media for which the formulae of Fresnel and Snell hold. If formula (18) had not agreed with the last statement an error in the algebraic work would have been indicated.

As will be shown below, in order that a minimum of reflecting power may exist the relative index of refraction must fulfill the conditions $0\leqq n<2-3^{1/2}$ and $\infty\geqq n>2+3^{1/2}$. The first and second ranges of values correspond respectively to partial internal reflection and to external reflection. As written, the first limits of these ranges are to be understood as reciprocal and the two binomial surds are reciprocal numbers, hence substitution in formula (18) shows that the minimum of reflecting power always lies between the extreme values 1/2 and 1/3, i.e., 50 per cent and 33⅓ per cent.

Attention will now be turned to the derivation of a formula for the calculation of $i_{\min}$. Equation (12) may be written as the following quadratic in y:

$$16xy^2+(x+1)Ty-xT=0 \tag{19}$$

where T denotes the trinomial

$$\left.\begin{aligned} &T\equiv x^2-14x+1, \\ \text{or}\quad &T\equiv n^4-14n^2+1. \end{aligned}\right\} \tag{20}$$

The general solution of equation (19) is

$$y=\{-(x+1)T\pm[T(x^2-6x+1)^2]^{1/2}\}/(32x) \tag{21}$$

The expression affected by the exponent $1/2$ shows that T cannot be negative and give real values of y since $x(\equiv n^2)$ is real. Therefore, since the expression affected by the power $1/2$ may be written $(x+1)^2T^2+64x^2T$ its square root is not less than the quantity $(x+1)T$ which is either positive or zero. Consequently the sign of the right hand member of equation (21) is controlled by the sign of the root chosen. But $y\equiv\sin^2 i_{\min}$ and, since this must not be negative, the positive root must be used. Furthermore, since x^2-6x+1 is identically $T+8n^2$ it is positive and hence it may be written in front of $T^{1/2}$ without influencing the sign of the quantity added to $-(x+1)T$ between the braces of (21). In short, equation (21) may be rewritten as

$$\sin^2 i_{\min}=\{[T+8n^2-(n^2+1)T^{1/2}]T^{1/2}\}/(32n^2) \tag{22}$$

$$=(2n^2T^{1/2})/[T+8n^2+(n^2+1)T^{1/2}]. \tag{22'}$$

Finally, since

$$(T+8n^2)^2-[(n^2+1)T^{1/2}]^2=(8n^2)^2,$$

which is the square of a rational function, it is possible to express the square root of the binomial surd included between the (square) brackets of equation (22) as another binomial surd. Consequently, the new final formula is

$$\sin i_{\min}=\{[n^2+1-|T^{1/2}|](|T^{1/4}|)\}/(8n) \tag{23}$$

$$=2n|T^{1/4}|/(n^2+1+|T^{1/2}|). \tag{23'}$$

Formula (23) must be incomparably simpler than the equation of degree 24 in $\sin i_{\min}$ which Jentzsch-Gräfe said would follow from the elimination of r or β (*vide supra*).

Since T, that is n^4-14n^2+1, may be written as

$$(n^2+4n+1)[n-(2+3^{1/2})][n-(2-3^{1/2})]$$

and since n cannot be negative, it should be clear that T is not negative

and hence i_{min} is real when, and only when, $0 \leqq n \leqq 2-3^{1/2}$ with $2+3^{1/2} \leqq n \leqq \infty$.

Let $u \equiv n+1/n$ so that $n^2+1/n^2 = u^2-2$, then substitution in formula (23) transforms it into

$$\frac{\sin i_{min}}{n^{1/2}} = \frac{1}{8}[u-|(u^2-16)^{1/2}|]\,|(u^2-16)^{1/4}| \tag{24}$$

$$= 2\,|(u^2-16)^{1/4}|\,/[u+|(u^2-16)^{1/2}|]. \tag{24'}$$

The right-hand member of formula (24), being a function of u only, does not change when n is replaced by $1/n$, that is, when (partially) internally reflected "light" is substituted for externally reflected light. This member is very convenient for numerical computation. Furthermore, Snell's law (1) gives $(n^{1/2})\sin r = (\sin i)/n^{1/2}$, hence the right-hand member of formula (24) also equals $(n^{1/2})\sin r_{min}$, or the direction of propagation may be reversed so that r_{min} and i_{min} may be looked upon as the angles of incidence and refraction respectively.

From formula (22) it is not difficult to derive the following expression for $\cos i_{min}$ which has stood the writer in good stead in theoretical work:

$$\cos i_{min} = 2(n^2+1)/\{\,|\,[v+8n(n^2+1)]^{1/2}|+|\,[v-8n(n^2+1)]^{1/2}|\,\} \tag{25}$$

where $v \equiv n^6-13n^4+19n^2+1$.

Differentiation of formula (23) with respect to n gives

$$\frac{di}{dn} = 2(n^2+1)(2n^4+17n^2-1)/(DT^{3/4}\cos i_{min}) \tag{26}$$

where $$D \equiv 2n^6-21n^4+8n^2-1+(2n^4-7n^2-1)\,|\,T^{1/2}|\,.$$

Substitution of the expression for $\cos i_{min}$ from (25) in (26) gives

$$\frac{di}{dn} = (2n^4+17n^2-1)\{\,|\,[v+8n(n^2+1)]^{1/2}| + |\,[v-8n(n^2+1)]^{1/2}|\,\}/(D\,|\,T^{3/4}|\,). \tag{27}$$

Hence i_{min} will have a stationary value (which plotting shows to be an algebraic maximum) when $2n^4+17n^2-1=0$. The only acceptable root of the last quadratic in n^2 is

$$n = \tfrac{1}{2}[3\,|\,(33)^{1/2}|-17]^{1/2} \doteq 0.2417064$$

Substitution of this value of n in formula (23) leads to

$$(\sin i_{min})_{max} = [7-(33)^{1/2}][9+(33)^{1/2}]^{1/2}/[16(2)^{1/2} \doteq 0.2130473$$

whence

$$(i_{\min})_{\max} \doteq 12°18'3.6''$$

The same substitution in formula (18) gives

$$R_{\min} = [19-(33)^{1/2}]/36 \doteq 0.3682066$$

Before presenting in graphical form the dependence of $i_{\min}$ and $R_{\min}$ on n it is desirable to find the values of these two quantities for certain other special values of n.

Formula (23′) shows directly that $i_{\min}=0$ when n equals 0, $2-3^{1/2}$, and $2+3^{1/2}$. By first dividing both numerator and denominator of (23′) by n^2 and then letting n approach infinity it will be found that $\sin i_{\min}=1$, that is, $i_{\min}=90°$. For $n=0$ formula (27) gives $di/dn=1$ so that the tangent to the curve at the origin bisects the angle between the axes of coördinates. The same formula shows that di/dn becomes infinite when $T=0$, that is, for $n=2-3^{1/2}$ and for $n=2+3^{1/2}$. By dividing both numerator and denominator of (27) by n^7 it will be seen that di/dn approaches $1/n^2$ as n becomes extremely large so that when n becomes infinite di/dn becomes zero of the second order, that is, the tangent to the curve becomes parallel to the axis of n.

Throughout almost all of the preceding analysis and discussion n has been treated as the independent variable. The dependence of $i_{\min}$ and $R_{\min}$ upon n is shown graphically in the accompanying figure. However, under certain circumstances it is convenient to have formulae [(28) to (32)] which give directly n and $i_{\min}$ or n and $R_{\min}$ when $R_{\min}$ or $i_{\min}$ are taken respectively as independent variables. Complete proofs of these formulae will be omitted and the subscript "min" is to be understood when not printed.

From (17) and (18) it follows that

$$n=\frac{|\,[2(1-R)]^{1/2}\,|\pm 1}{|\,(1-2R)^{1/2}\,|}=\frac{|\,(1-2R)^{1/2}\,|}{|\,[2(1-R)]^{1/2}\,|\mp 1} \tag{28}$$

Substitution from (28) in (23) or in (23′) leads to

$$\sin i_{\min}=[2(3R-1)]^{1/4}\{\,|\,[2(1-R)]^{1/2}\,|\pm 1\}^{1/2}/[\,|(1-R)^{1/2}|+|(3R-1)^{1/2}\,|\,] \tag{29}$$

Formula (12) solved for x or n^2 becomes the cubic

$$(1-y)x^3-(14-13y)x^2+(1+13y-16y^2)x-y=0 \tag{30}$$

in which $y\equiv\sin^2 i$.

From (28) may be obtained

$$x^2-2ex+1=0 \quad , \tag{31}$$

where $e\equiv(3-2R)/(1-2R)$.

Elimination of x between (30) and (31) leads eventually to the following cubic in $R_{\min}$

$$2a_0R^3-a_1R^2+4a_2R-a_3=0 \tag{32}$$

where

$$a_0\equiv 9-36y+60y^2-48y^3+16y^4,$$
$$a_1\equiv 21-120y+232y^2-176y^3+48y^4,$$
$$a_2\equiv 2-14y+31y^2-24y^3+6y^4,$$
$$a_3\equiv 1-8y+20y^2-16y^3+4y^4.$$

For sake of completeness it seems desirable to discuss very briefly the dependence of the minimum reflecting power upon the index of refraction when this index lies between the limits $2-3^{1/2}$ and $2+3^{1/2}$. Equation (5) shows that this minimum occurs at normal incidence, that is, when $i=r=0$, hence the value of the reflecting power will be denoted by R_0. From (1), (2), (3), and (4) the very familiar formula (33) follows

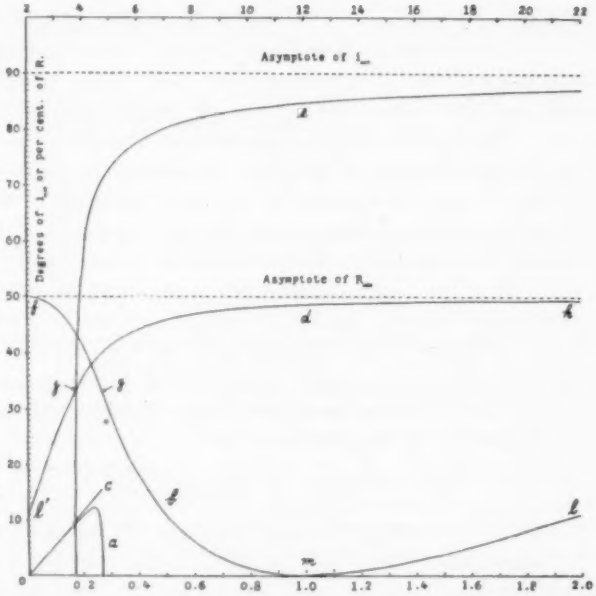
$$R_0=\left(\frac{n-1}{n+1}\right)^2 \tag{33}$$

For $n=2\pm 3^{1/2}$ formulae (18) and (33) give $R_0=R_{\min}=1/3$. For $n=2-3^{1/2}$ $dR_0/dn=dR_{\min}/dn=-2(2\cdot 3^{1/2}+3)/9\doteq -1.436467$. For $n=2+3^{1/2}$ $dR_0/dn=dR_{\min}/dn=2(2\cdot 3^{1/2}-3)/9\doteq 0.1031337$. It is thus clear that the curves representing equations (18) and (33) have the same ordinates and common tangents at the points whose abscissas are $2-3^{1/2}$ and $2+3^{1/2}$, respectively. Accordingly, to this degree of accuracy, the minimum of reflecting power suffers no discontinuity when n passes through the values $2-3^{1/2}$ and $2+3^{1/2}$. Obviously the angle of incidence $i_{\min}$ does change abruptly its rate of variation with n since it maintains the value zero from $n=2-3^{1/2}$ to $n=2+3^{1/2}$.

On the other hand, when $n=2-3^{1/2}$ $d^2R_0/dn^2=2(7\cdot 3^{1/2}+12)/9\doteq 5.360968$ while $d^2R_{\min}/dn^2=-(39+22\cdot 3^{1/2})/9\doteq -8.5672353$. Hence inflection is simulated at the point $(2-3^{1/2},\ 1/3)$ where the curves for R_0 and $R_{\min}$ meet. When $n=2+3^{1/2}$ $d^2R_0/dn^2=-2(7\cdot 3^{1/2}-12)/9\doteq -0.0276346$ whereas $d^2R_{\min}/dn^2=-(39-22\cdot 3^{1/2})/9\doteq -0.0994314$ and so the curvature of the two curves is in the same direction.

Finally as regards the graph of equation (33) it should be observed that for $n=1$ $R_0=0$, $dR_0/dn=0$, and $d^2R_0/dn^2=1/2$, that is an absolute

minimum of reflecting power occurs when the speed of light has the same value in the two media. When $n=2$ $R_0=1/9$ or 11.11 per cent. $dR_0/dn=4/27$, $d^2R_0/dn^2=0$, and $d^3R_0/dn^3=-8/81$, that is a true point of inflection occurs.



The salient features of the various functions are shown in the figure. The scales of abscissas for curves a, b, c and for curves d and e are indicated respectively along the lower and upper "horizontal" lines of the diagram. Curve a represents the dependence of $r_{\min}$ upon n. It necessitated relatively great magnification along the axis of n, since 0.268 is the greatest value of this index associated with $r_{\min}$. A segment of the graph for the critical angle is shown near c. The arc fg pertains to $R_{\min}$ from $n=0$ to $n=2-3^{1/2}$. The variation of R_0 as far as the true point of inflection l is given by the arc gml. The graph of R_0 is continued (on a different scale) from l' to j. At $j, n=2+3^{1/2}$ so that the arc jk pertains to $R_{\min}$ from this value of n to infinity. The curve e represents $i_{\min}$ over the same range of n, that is, $2+3^{1/2} \leqq n \leqq \infty$.

YALE UNIVERSITY,
NEW HAVEN, CONNECTICUT.
MARCH 1, 1927.

JOINT MEETING OF THE OPTICAL SOCIETY OF AMERICA AND THE AMERICAN PHYSICAL SOCIETY IN NEW YORK CITY, FEB. 25–26, 1927

A joint meeting of the Optical Society of America and the American Physical Society was held in New York City at Columbia University on February 25 and 26. The sessions were held in the new Physics Laboratories on 120th Street and marked the formal opening of these laboratories. The Optical Society papers were presented on Friday, those of the Physical Society on Saturday. In addition to the inspection of the laboratories, special features of the meetings were the invited papers by Professor F. K. Richtmyer on "The Extension of the Classical Laws of Optics into the X-Ray Region of the Spectrum" and by Professor R. W. Wood on "The Experimental Control of the Radiating States of Optically Excited Atoms." A complimentary luncheon was served by the University on Saturday and enjoyed by all. There follows the abstracts of the papers presented at the sessions of the Optical Society. The abstracts of the Physical Society papers will be found published in the Physical Review.

CHARLES C. BIDWELL, SECRETARY,
OPTICAL SOCIETY OF AMERICA

1. Deane B. Judd — **Munsell Research Laboratory, and Bureau of Standards**

PURITY AND SATURATION; A SATURATION SCALE FOR YELLOW

The relation between purity and saturation, with hue constant, known as a saturation scale, has been evaluated by Nutting and Jones[1] for three hues (red, green, and blue); later by Jones and Lowry[2] for hues corresponding to eight dominant wave lengths; and still later for red at the Munsell Research Laboratory by unpublished work involving many observers. The results of these investigations are hard to reconcile in all particulars, but they concur in indicating that: (1) the relation, $Purity = Constant \times Saturation$, is approximately correct; and (2) the character of the relationship between purity and saturation is nearly independent of hue.

If we assume that the saturation attached to a visual stimulus by an observer rests upon that observer's color vision and not upon the vagaries of his judgment as determined by education, dyspepsia, etc., the saturation of a visual stimulus may be computed from its dominant wave length and purity, and the relation between purity and saturation is thereby determined. If we take the O. S. A. Excitations as fairly representing average color vision, the following statements are true concerning the relation between purity and an evaluation of saturation into which errors essentially psychic do not enter: (1) Both statements given above and derived from experimental data are correct; (2) for the violet, the deviation of saturation from a straightline relation with purity is very marked; the ratio of the purity increment at purity, 0.9, to the increment at purity, 0.1, is as high as 100.0; (3) and, in general, this ratio is equal to $S(36S+3)/(3S+4)$, where $S = RL_r + GL_g + BL_b$, R, G, and B being the trilinear coördinates of the spectrum, and L_r, L_g, and L_b being the luminosity coefficients attached to the color primaries of the O. S. A. Excitations.

An experimental procedure has been devised whereby the shape of the curve between purity and saturation may be found without the use of liminal determinations. For dominant wave length about 575 mμ, the relation has been determined by two observers. The actual colors corresponding to this saturation scale have been compared by seven other observers with the colors specified for dominant wave length 575 mμ by the workers above referred to. There has resulted a close check on the saturation scale derived from the O.S.A. Excitations, a much closer check, in fact, than that yielded for any of the previously determined experimental scales.

Bibliography:
(1) Nutting, J.O.S.A., *4*, p. 62, 1920.
(2) Jones and Lowry, J.O.S.A. & R.S.I., *13*, p. 25, 1926.
The present paper will appear in full in J.O.S.A. & R.S.I.

2. Walter H. Stainton **Cornell University**

THE PHENOMENON OF BROCA AND SULZER IN FOVEAL VISION AS DEPENDENT UPON WAVE LENGTH AND INTENSITY

Two luminous fields balanced photometrically will not, in general, appear equal when one of them is exposed for a fraction of a second. By repeated trials, however, the field continuously exposed may be adjusted in brightness to match the apparent luminosity of the flashed field. A series of such matches for various durations of flash has been used by Broca and Sulzer and others to obtain a curve of apparent luminosity as a function of time.

An apparatus has been arranged to use a method essentially that of Broca and Sulzer. A fairly complete set of curves has been obtained for one observer over a wide range of brightnesses.

It is found that the "overshooting" effect appears in the green at brightness levels substantially the same as those for white. The high maxima of Broca and Sulzer for the blue are absent with small fields, as has also been shown by Kleitman and Piéron. At low brightness levels, the maximum is later and less clearly defined than is indicated by the work of Bills. For the light adapted eye, the rate of rise is greater than for dark adaptation at corresponding brightnesses.

Bibliography:
Broca and Sulzer: C.R., *137*, pp. 944, 977; 1903.
Bills: Psychol. Rev. Monogr., *28*, No. 5, 1920.
Kleitman and Piéron: L'Année Psychologique, *25*, p. 34, 1924.
The present paper will appear in full in J.O.S.A. & R.S.I.

3. Theodore Lyman **Harvard University**

THE GAP IN THE SPECTRUM

The paper contains a review of the recent attempts to investigate the radiations lying in the region between soft x-rays, λ20, and the extremest ultraviolet, λ200.

It is pointed out that the indirect method in which the photoelectric action of soft x-rays is employed yields results which cannot be easily interpreted in this region. Papers of O. W. Richardson[1] and K. T. Compton[2] are cited in this connection.

The investigations of Thibaud[3] with a glass grating and of Dauvillier[4] with a crystal lattice are discussed in the light of some recent work of the author.

Bibliography:
1. O. W. Richardson, Proc. Roy. Soc., *110*, p. 247; 1926.
2. K. T. Compton, Physical Rev., *28* p. 601; 1926.
3. Jean Thibaud, Jour. de Phy. Jan. 1927.
4. A. Dauvillier, Jour. de Phy. Jan. 1927.

4. N. Rashevsky **Research Dept., Westinghouse Electric and Manufacturing Co.**

LIGHT-QUANTA AND MAXWELLS EQUATIONS

In a recent paper[1] N. Kasterin attempted to show, that the model of a light-quantum, proposed by Sir J. J. Thomson, namely that of a closed tube of force moving with the velocity of light[2] may be obtained as a solution of Maxwells Equations, provided we admit also discontinuous solutions.

In the present paper it is pointed out that any discontinuity of the field behaves *formally* either like a charge or a current. And since we can observe and measure charges and currents *only by their action*, that is, by their *behaviour*, the assumption of discontinuities of the field, necessary to give a picture of J. J. Thomson light-quantum, leads to the conclusion that such a light-quantum carries not only energy, but also charge, or, if it is electrically neutral, a system of currents, suitably distributed. In other words, we must ascribe to the quantum a complicated internal structure, which conclusion is not exempt of certain difficulties.

A further investigation shows, however, that although the solution, proposed by N. Kasterin cannot be regarded as quite satisfactory, there is still no reason to believe in a necessary contradiction between the conception of directed quanta and Maxwells Equations. Even considering only such solutions which are represented by retarded potentials, we obtain radiation in spherical waves only when certain restricting conditions are implied. It is shown that in a general case the possibility of a directed radiation is not excluded also for these solutions.

Bibliography:
1. Phil. Mag. [7] II, p. 1208, 1926.
2. Ibid. [6] XLVIII, p. 737, 1924 and I, p. 1182, 1925.

5. Ludwik Silberstein and Clifton Tuttle **Research Laboratory, Eastman Kodak Co.**

THE RELATION BETWEEN THE SPECULAR AND THE DIFFUSE PHOTOGRAPHIC DENSITIES

The purpose of the paper is to derive theoretically a formula correlating the so-called specular and diffuse density of a layer of a light-scattering medium, such as a developed photographic film or plate, and to test it experimentally. In accordance with the actual method of measurement of the two kinds of "density," the specular density (D) is defined as the common logarithm of the ratio of the intensity of the incident light to that of the beam transmitted normally to the layer, which will include some fraction (ϵI_s) of the scattered light, and the diffuse density (D') as the logarithm of the ratio of the intensities of the incident and the total transmitted light including the whole scattered light. The incident rays are in both cases supposed to be collimated normally to the layer.

It is assumed that of the direct (collimated) light, of intensity I, penetrating to any depth each silver grain scatters the amount AI and absorbs $\kappa\omega I$, where ω is the average effective area of a grain, κ a numerical factor, and A a coefficient depending on the wave length and the size of the grain. Also part of the scattered light (I_s) produced within the layer is absorbed by the grains, and as these show no trace of orientation, the amount absorbed by a grain is again expressed by $\kappa\omega I_s$, i.e., with the same coefficient as for the direct light. The effect of an "infinitesimal" stratum with regard to both scattering and absorption is taken to be proportional to the number of grains contained in it. Under these circumstances the finally emerging intensities I and I_s are found at once by solving a pair of simple differential equations. Whence follow the densities $D=\text{Log}\,[I_0/(I+\epsilon I_s)]$ and $D'=\text{Log}\,[I_0/(I+I_s)]$ as functions of the total number (n) of grains per unit area of the layer. Eliminating n, the required relation between the two densities is found to be

$$10^{-D}=\epsilon\cdot 10^{-D'}+(1-\epsilon)\cdot 10^{-\beta D'},$$

where $\beta=1+A/\kappa\omega$ and the significance of the fraction ϵ is as above. Equivalently, in terms of transparencies, $T=\epsilon T'+(1-\epsilon)T'^{\beta}$

A devise for determining the constants ϵ, β from two observed pairs of corresponding D, D' being given, the formula is tested on the experimental results obtained with five different kinds of emulsion representative of various grain sizes, and is found to give in each case a close agreement.

The present paper will appear in full in J.O.S.A. & R.S.I.

6. Thornton C. Fry **Bell Telephone Laboratories, Inc., New York City**

Introduced by H. E. Ives

LIGHT WAVES IN METALS

When a wave of plane polarized light falls obliquely upon a conducting surface, it gives rise to a disturbance inside the conductor which has, among others, the following peculiarities: (a) It is neither plane nor elliptically polarized, but belongs to a third distinct category; (b) it does not travel with what is customarily called "the speed of light"; (c) its velocity varies with the angle of incidence.

There are similar light waves in dielectrics and in free space.

7. Lauriston Taylor **Cornell University**

A COMPARISON OF THREE METHODS OF SPECTROPHOTOMETRY OF WEAK LIGHT

A comparison of sensitivity and precision was made for the three spectrophotometric methods employing the following instruments: (1) Lummer-Brodhun spectrophotometer, (2) a spectrograph in which the comparison spectra were photographed simultaneously by means of a divided slit, (3) a photoelectric spectrophotometer. The comparisons were made for intensities of light near and below the lower limit of visual observation. In the photographic method, several different plates were used. The last method employed a photo-electric cell in place of the eye and was used with a large aperture spectrometer. A high sensitivity Compton electrometer was used to measure the photo-electric current. For light that could be seen only after dark adaptation, the error of observation by method (1) was of the order of 16 per cent. With 1/200 of the above intensity the error by method (2) was 5 per cent to 25 per cent depending upon the plate used and the time of exposure. This method required exposures from 2-20 hours in length for a single comparison. For the same intensity (1/200) of light, the error by the photoelectric method was 2 per cent to 6 per cent, and the time required for an obseration was about 5 minutes or less. The time required for a set of six observations with corresponding dark drifts was about 30 minutes. The chief factors which govern the precision and sensitivity of this method are: the type and sensitivity of the electrometer and the potential applied to the cell. There is a considerable gain in precision at the expense of sensitivity.

The present paper will appear in full in J.O.S.A. & R.S.I.

8. C. W. Woodworth **University of California**

THE WAVE FRONT IN THE FOCUS OF A CORRECTED LENS

A new method for calculating the optical length of a ray of light passing through a lens is presented together with examples of the calculation of a series of rays through the great 36-inch telescope objective at Lick Observatory.

The purpose of these calculations is to determine with extreme accuracy the character of the wave front in the focus of a highly corrected lens. It is well known that in an uncorrected lens the wave front passes through a very complicated transformation (Microscope

Theory, pp. 56–59) and because of the unavoidable zonal aberrations one could assume that there were comparable transformations in the focus of a corrected lens which proves to be true.

This is an important matter in the theory of diffraction where one considers differences of half wave lengths. It has been previously assumed in the discussion of diffraction that the wave front approached the focus as a spherical surface.

Bibliography:
Woodworth, C. W., Microscope Theory, Trans. Science Soc. of China, pp. 29–230; 1924.
The present paper will appear in full in J.O.S.A. & R.S.I.

9. Christine Ladd-Franklin **Columbia University**

PROOF THAT NERVE WHEN EXCITED, AND PROBABLY ALSO WHEN NOT EXCITED, EMITS PHYSICAL LIGHT

There is an entoptic phenomenon—what I have called the *Reddish-Blue Arcs and the Reddish-Blue Glow of the Retina*—in which it is easily shown that what you are seeing is (1) certain excited nerve fibers which proceed on the surface of the retina to the exit point of the optic nerve (the papilla) and (2) the point-like bipolar cells which lie around the image of the red band.* But why is an excited nerve fiber visible? The theory that has been given of this phenomenon (by Gertz and others) is that an excited nerve fiber will produce an induced electric current in adjoining fibers, and through them make itself visible. But it is plain that such a nerve excitation as this would not reach the cortex with the required "place-coefficients,"—when a leg has been amputated, pinching the cut nerve fibers does not give a sensation located at the place where they are pinched but at the toes or the heel. Nothing but light will cause an object to be seen at the place where it is. It follows that in this case the excited nerve fiber is actually emitting physical light. But absolute proof of this view is furnished by the fact that these reddish-blue arcs give an after-image, and that after-images do not occur in the case when a nerve only is excited: the highly specific form of exhaustion in the light-sensitive substance of the rods and cones is essential to the production of the after-image.

But if all nerve when in a state of activity emits physical light (or some radiation of higher frequency which immediately becomes physical light through fluorescence), why has this not been discovered long since? There are two reasons for this: most work on nerves is done on myelinated nerves—those in the retina are non-myelinated and consequently let light through; one does not usually study the nerve in a perfectly dark room—what is necessary to seeing this reddish-blue light.

Shiro Tashiro and G. H. Parker have shown that carbon dioxide is given off even by unexcited nerve. Hence light also may be emitted by unexcited nerve.

* Point-like when seen in cross section.

Bibliography:
1. Lasareff, P., Comptes rendus, *178*, p. 1100; 1924.
2. Nodon, A., Comptes rendus, *178*, 486; 1924.
The present paper will appear in full in the English psychological journal, Psyche.

10. Henry J. Miller **Research Laboratory, General Elec. Co., Nela Park**

ABSORPTION SPECTRA OF SOME ORGANIC DYESTUFFS AND THE USE OF THESE DYESTUFFS FOR "MONOCHROMATIC" LIGHT-FILTERS IN THE ULTRAVIOLET PART OF THE SPECTRUM

The absorption spectra of the following dyestuffs were investigated:

1. p-Nitroso-ethyl-aniline (Merck)
2. p-Nitroso-diethyl-aniline (M)
3. 5-Diethylamino-2-nitrose-phenol (Kahlbaum)
4. p-Nitroso-ethyl-aniline hydrochloride (Kahlbaum)

5. p-Nitroso-diphenylamine (Kahlbaum)
6. p-Nitroso-methyl-aniline (M)
7. p-Nitroso-dimethyl-aniline (By)
8. p-Nitroso-dimethyl-aniline (Merck)
9. Fuchsine (blaust, M)
10. Uranine
11. Diamond fuchsine (By)
12. China green (By)

These dystuffs were diluted in water and ethyl alcohol and the absorption spectra were observed as a function of degree of dilution. It was found that, for example, p-Nitroso-dimethyl-aniline (By) and Fuchsine (blaust, M) have two absorption bands, which are covering the entire spectrum of the mercury vapor arc, when the two solutions are combined to a double filter, transmitting only the ultraviolet lines from 3900 Å to 2950 Å. Therefore a suitable filter transmitting only ultraviolet radiation can be made out of these dyestuffs. The same principle for obtaining suitable "monochromatic" light filters can be applied for numerous other dyestuffs and for all parts of the light spectrum.

Bibliography:

1. W. N. Hartley, Relations between absorption and constitution of organic compounds in H. Kayser, Handbuch der Spectroscopie, *3*, 1905.

2. Jaroslav Formanek, Untersuchung und Nachweis organischer Farbstoffe auf spectroscopischem Wege, Berlin 1908–1926.

11. Deane B. Judd **Munsell Research Laboratory, and Bureau of Standard**

THE EMPIRIC RELATION BETWEEN DOMINANT WAVELENGTH AND PURITY

It has long been known that the same hue might not be attached to two visual stimuli of the same dominant wavelength but of different purities. Although Abney[1] made quantitative measurements of the "hue shift," his results are rendered somewhat uncertain by doubt as to the true "neutrality" of his standard "white."

Observations incidental to a protracted series of color analyses by the monochromatic method have resulted in a qualitative check of Abney's findings, and with a standard "neutral" that is completely specified. The observations which refer to dominant wavelengths greater than 500mμ have been accounted for on the basis of the well-known saturation changes with wavelength at constant purity, as follows:

(1) For dominant wavelengths between 500 mμ and 570 mμ, a decrease in the purity oft he mixture field results in a lower experimental dominant wavelength for the sample.

(2) For dominant wavelengths between 570 mμ and 640 mμ, a decrease in the purity of the mixture field results in a higher experimental dominant wavelength for the sample.

In these cases, the range of possible values for dominant wavelength of the sample is largest for those wavelength regions in which sensibility to wavelength change is smallest, and the direction of the variation of dominant wavelength with purity is such that two fields of slightly different hue are reacted to as matched because the saturations they evoke are equal, the hue difference being too slight to be perceived.

If the dominant wavelength of the sample is less than 460 mμ, a reduction of the purity of the mixture field makes the field appear more violet than before and leads to an increase in the experimentally determined dominant wavelength of the sample. A change toward this higher wavelength does not tend, however, to equate the saturations of the two fields; hence the explanation which holds for the long wavelengths fails for the short. No simple alternative explanation seems to exist.

Bibliography:

(1) Abney, On the Change of Hue of Spectrum Colors by Dilution with White Light, Proc. Roy. Soc., A., *87*, 1909.

The present paper will appear in full in J.O.S.A. & R.S.I.

12. **P. V. Wells and O. E. Conklin** **Redpath Laboratory, Dupont-Pathe Film Mfg. Co., Parlin, N. J.**

THE MENTAL FACTOR IN VISUAL ACUITY

In testing the visual acuity of applicants for the plant, the measurements revealed, in addition to the retinal acuity, a mental factor. This factor could not be eliminated from the criterion of acuity, indeed it appears to be an attribute essential to all measures of sensations. The paper describes a method for determining visual acuity which is free from subjective bias, and presents results which show the importance of the mental factor.

Black broken circles of geometrically graded size on white paper are successively placed before the subject for three seconds at a time. The break may be in one of four positions, top, left, right or bottom. The answers are corrected for the fact that one quarter of the positions can be guessed correctly. The acuity criterion is the least angular opening resolved in one half the trials. Long after the subject has become fully accommodated to the low level of illumination (safe for cine-negative for 30 minutes), and in spite of frequent rest periods, the acuity can apparently double because of mental changes in the observer. Extreme caution, therefore, must be exercised not to grade operators falsely because of accommodation, fatigue, and psychological disturbing factors.

The present paper will appear in full in J.O.S.A. & R.S.I.

Apparatus for the analysis of feebly polarized light.—Fringe polariscopes, such as the Savart, are not capable of detecting a ratio of polarized light less than 1/100, but the sensitivity may be increased by combining the polarization to be studied with that of an inclined plate of glass. A polarimeter of this kind, intended for astronomical work especially on the light of the planets, is described. The measurement of the amount of polarized light when it is 1/1000 of the whole was found to be possible. With another polarimeter, utilizing the same principles but intended for laboratory use, an ellipticity of 1/20 could be detected when the proportion of polarized light was 1/100.

Percentage of polarization in the light reflected from various solids was found to change rapidly with the angle of separation of the source and the polarimeter as seen from the reflecting or diffusing surface. Observations of the moon gave results showing a reversal of sign of the polarization at about 24° with minimum and maximum at about 10° and 100°. Mercury from 60° to 130°, and Mars, from 0° to 45°, gave similar results; but for Venus, Jupiter and Saturn the curves were entirely different. The curve for Venus shows four inversions while for Jupiter and Saturn the polarization of the polar regions is independent of the relative positions of the sun and the earth with respect to the planet. The percentage polarization is a maximum at the poles, reaching 6% on Jupiter where the extreme polar value is constant. On Saturn, the polar value sometimes changes considerably.—[Bernard Lyot; Revue d'Optique, *5*, pp. 109–118; 1926.]

G. W. MOFFITT.

FINANCIAL REPORT

Journal of the Optical Society of America and Review of Scientific Instruments
for the Period from January 1, 1926 to December 31, 1926

RECEIPTS

Cash on Hand, January 1, 1926		$ 3,213.65
To Apply on Contract of the Optical Society		
Subscriptions	$ 458.30	
Sale of Back Numbers of Volumes I–V	301.50	
Advertising Collected for Optical Society	714.34	
Payments from Treasurer of Optical Society	3,480.40	
Sale of Reports of Committee on Colorimetry	5.00	
		4,959.54
To Apply on Contract of the Association of Scientific Apparatus Makers of the United States of America		
Advertising		7,844.94
Miscellaneous Receipts		
Subscriptions	3,478.88	
Sale of Back Numbers of Volumes VI–13	440.97	
On Account of Helmholtz Translation (contra)	18.90	
Reimbursement-Expenses in Connection with Montreal meeting.	4.00	
Overpayment on Colorimetry Report	.50	
Total		3,943.25
Grand Total		$19,961.38

DISBURSEMENTS

Printer	$12,295.19	
Salaries	4,448.25	
Supplies	133.03	
Miscellaneous (Postage, telegrams, etc.)	478.36	
		$17,354.83
Cash on Hand, December 31, 1926		2,606.55
		$19,961.38

The above is a correct statement of accounts to the best of my knowledge and belief.

(Signed) F. K. RICHTMYER, *Business Manager*

We, the undersigned, duly appointed to audit the accounts of the Journal of the Optical Society of America and Review of Scientific Instruments as at the close of business, December 31, 1926, hereby certify that we have examined the statement of receipts and disbursements, the vouchers, check book and bank book and find them to agree with each other. We, therefore, certify that the above statement is a true statement of the accounts of the Journal of the Optical Society of America and Review of Scientific Instruments as of December 31, 1926.

(*Signed*) C. C. BIDWELL
LEO BEHR
Auditors

OPTICAL SOCIETY OF AMERICA
STATEMENT OF CASH RECEIPTS AND DISBURSEMENTS

For the Period from January 1, 1926 to December 31, 1926

CASH ON HAND, JANUARY 1, 1926		$10,038.49
RECEIPTS:		
Dues	$2,922.00	
Donations	3,000.00	
Interest on Bank Balances	333.14	
Journal Subscriptions on Books Prior to February 15, 1922	452.55	
Advertising	714.34	
Sale of Back Numbers	301.50	
Sale of Colorimetry Reports	5.00	
Miscellaneous	.15	
Total Receipts		7,728.68
		$17,767.17
DISBURSEMENTS:		
Printing and Stationery	$ 178.50	
Postage	40.36	
Expenses of Meetings	178.48	
Journal Subscriptions since February 15, 1922	141.00	
Association of Scientific Apparatus Makers of United States Contract	4,800.00	
Miscellaneous	7.08	
Total Disbursements		$ 5,345.42
CASH ON HAND, DECEMBER 31, 1926		12,421.75
		$17,767.17

I certify that the above report is correct to the best of my knowledge and belief.

ADOLPH LOMB,
Treasurer

Rochester, New York
January 25, 1927

Adolph Lomb, Treasurer,
Optical Society of America,
Rochester, New York.

Dear Sir:

In accordance with your request, we have audited the books of the Optical Society of America for the twelve months ended December 31, 1926.

We hereby certify that the above Statement of Cash Receipts and Disbursements is a true statement for the period from January 1, 1926 to December 31, 1926.

Respectfully submitted,
NARAMORE, NILES & CO.
Auditors

INSTRUMENT SECTION

AUTOMATIC GAS ANALYSIS RECORDER FOR THE RANGE OF 0 TO 3.5 PER CENT OF CO_2 IN AIR

By Crandall Z. Rosecrans

ABSTRACT

An automatic instrument for recording the percentage of carbon dioxide in air supplied to greenhouses for experimental work is described. The instrument gives a record over the range of 0–3.5% carbon dioxide. The application of the thermal conductivity method of gas analysis is described, and several new features not heretofore found in such apparatus are discussed.

Convection Type Thermal Conductivity Cell.—A cell construction is described which renders the indications of gas concentration independent of the velocity of flow through the sampling line. The circulation of gas through the cell is dependent on the platinum wire temperature only.

Compensation for Variations in Bridge Current.—A method is described for compensating the thermal gas analysis Wheatstone bridge for variations in the bridge current by introducing a manganin resistance in series with one of the cell wires.

Compensation for Variations in Cell Temperature.—A method is described for compensating the bridge for variations in cell temperature by introducing a nickel resistance in one of the bridge arms.

The results obtained with this instrument indicate that it is accurate to 0.05% of carbon dioxide, over the range of 0–3.5% carbon dioxide. The application of similar gas analysis bridges and recorders to the analysis of other gas mixtures where the same range of thermal conductivity differences is involved is discussed.

The general method of gas analysis utilizing the variation of thermal conductivity of different gases is well known. The application of automatically recording devices to such thermal conductivity apparatus, however, has in the past been limited to wide range instruments, as, for example, instruments recording variations of carbon dioxide in boiler flue gases, over the range of 0 to 20 per cent carbon dioxide.

The apparatus here described is of much greater sensitivity than those heretofore produced, and in addition embodies several new features which tend to increase the accuracy of determination.

Briefly, the thermal conductivity method for gas analysis is as follows. Two fine platinum wires are placed axially in two tubes of metal and insulated from the metal. One tube, or cell, is sealed tightly, with dry air in it. The other cell is connected so that the gas to be analyzed passes through it, and over the wire. The two wires are

connected in a Wheatstone bridge circuit as shown in Fig. 1. *S* is the sealed wire, or standard, and *X* is the wire exposed to the gas to be analyzed. *A* and *B* are the other arms of the Wheatstone bridge, and *K* is a slide wire for adjusting the bridge. If current from the battery *E* is passed through the bridge network, the fine platinum wires are heated. Assume that dry air is passed through the cell *X*, and that the bridge is balanced by an adjustment of the slide wire. Now if a gas of lower thermal conductivity than air (carbon dioxide, for example) is passed through the *X* cell, the temperature of the *X* wire will increase, as the heat will be conducted away from the wire at a lower rate, due to the lower thermal conductivity of the carbon dioxide. As the temperature of the *X* wire increases, its resistance also increases,

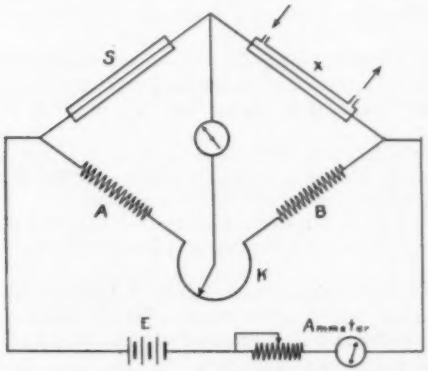


FIG. 1. *Theoretical thermal conductivity bridge circuit.*

and the bridge will be unbalanced. The slide wire *K* is then adjusted to rebalance the bridge. Thus it is evident that a calibration of the apparatus could be made, using the variation of the balance point on the slide wire *K* as an index of the percentage of CO_2 passing through the cell *X*.

It is of interest to determine the actual range of thermal conductivity difference over which the instrument must operate, since apparatus could be constructed in a similar way for the analysis of many other gaseous mixtures, where the same difference of thermal conductivity appeared.

The following calculations show the variation of thermal conductivity for the ordinary carbon dioxide recorder for boiler plants, having a range of 0 to 20 per cent carbon dioxide.

Gas	*Concentration*	*Thermal Conductivity**	*Product*
Air	1.00	0.0000719	0.0000719
CO_2	0	0.0000496	0
	thermal conductivity of mixture	=	0.0000719
Air	0.80	0.0000719	0.0000575
CO_2	0.20	0.0000496	0.0000099
	thermal conductivity of mixture	=	0.0000674

The difference of thermal conductivities for the two mixtures is 0.0000045. The apparatus described here has a range of from 0 to 3.5 per cent of CO_2, the remainder being air. It is evident that this

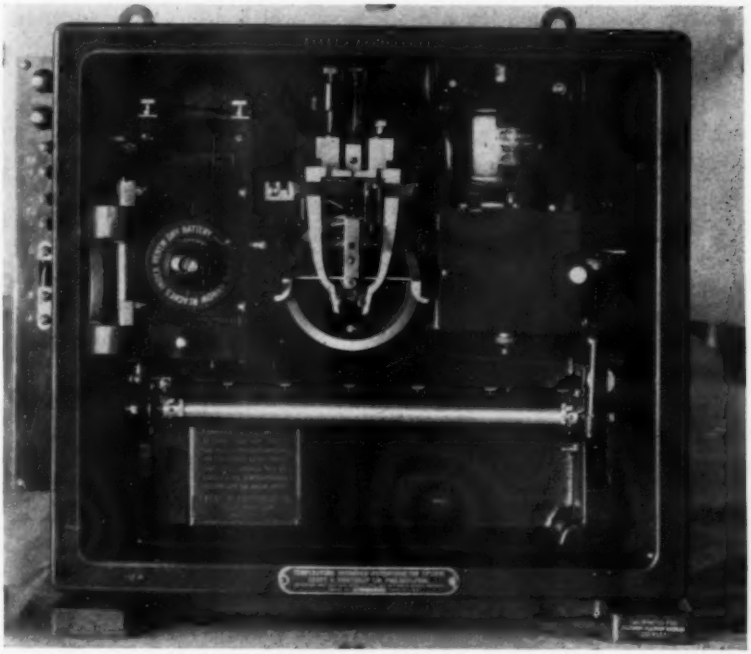


FIG. 2. *Front view of recorder.*

instrument must be almost 6 times as sensitive as the carbon dioxide recorder with the 0 to 20 per cent range, and must operate over a range

* All thermal conductivities are taken as those given by Eucken in the Physikalische-Chemische Tabellen. These values were determined at 100°C.

of thermal conductivity of 0.00000079. That is, the instrument described would operate satisfactorily for the analysis of a mixture of two gases, where the difference of thermal conductivity between the mixtures at the extreme ends of the scale range was 0.00000079. The apparatus is therefore not limited to the analysis of carbon dioxide and air mixtures, but is applicable to many different gas mixtures, where the thermal conductivity of the mixture varies over approximately the same range.

RECORDER

The recorder used is a self-balancing Wheatstone bridge, in which the galvanometer, when deflected by a current passing through it, operates to set in motion a mechanism which alters resistance, rebalancing the bridge. Fig. 2 shows a front view of the type of recorder

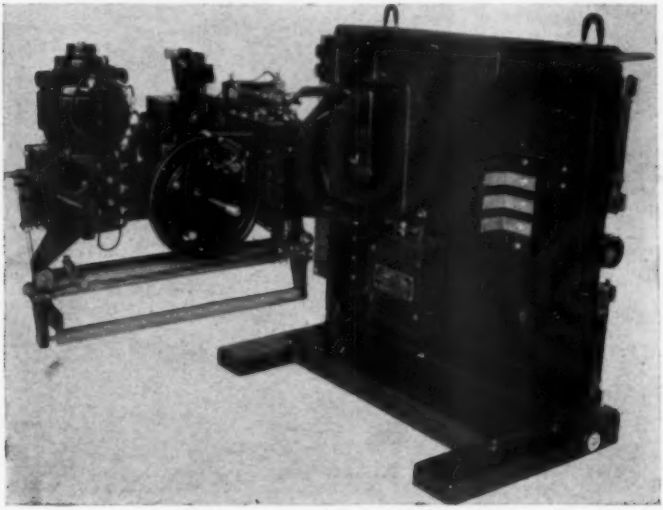


FIG. 3. *View of working parts of recorder.*

employed, and Fig. 3 a view of the rear working parts. Referring to Fig. 1, which shows the theoretical circuit, the bridge is balanced by adjusting the slide wire K. In the recorder the slide wire is carried on a disc, which may be seen in Fig. 3. The contact on the slide wire is fixed, while the disc is turned under the contact by the action of the recorder mechanism.

A small cable is wrapped around the slide wire disc and, passing over pulleys at the ends of the recorder frame, operates a pen carriage. Thus when the bridge is balanced by the action of the recorder, the pen indicates on the chart the position of the contact on the slide wire, and consequently, if the instrument has been properly calibrated, the concentration of carbon dioxide flowing through the cell *X*.

GAS ANALYSIS CELL

The gas analysis cell consists essentially of two cylindrical tubes, through which the platinum wires are stretched. The cell as actually used is shown diagrammatically in Fig. 4. The two tubes *S* and *X* are formed by drilling holes in a solid block of metal, and are 0.375″ in diameter and 4½″ in length. The wires are sealed into the tubes at the top by glass-platinum seals. Heavy platinum wire is used as lead-in wire, and the fine platinum wire 0.002″ diameter is gold soldered to the lead-in wires.

The outstanding feature of the cell as shown in Fig. 4 is the method of admitting gas to the *X*, or analyzing, wire. The gas sample is passed continuously downward through the tube *C*. A small portion of the sample passes through the tube *A* into the gas cell *X*. Here it is heated by the fine platinum wire (which attains a temperature of approximately 140°C), and consequently rises. It passes out of the cell at the top, through the diagonal tube *B*, and re-enters the main gas sample tube *B* at a point opposite the opening *A* through which it entered the cell *X*. The action of the gas is thus a by-passing of a small amount of the sample flowing in the tube *C*, through the analyzing cell *X*. This flow of gas is produced by the difference in weight of the two columns of gas, one in the cell *X* at high temperature, and one in the tube *B* at room temperature. The essential fact, however, is that the tubes *A* and *B* join the main gas sample tube *C* opposite each other. Thus there is no pressure difference existing between the inlet and the exit of the cell *X*, and the flow through the cell *X* is dependent only on the platinum wire temperature, being practically independent of the velocity of gas flow through the tube *C*. It has been found that the CO_2 recorder gives constant indications when a definite gas mixture is passed through the tube *C* at rates ranging from 5 to 150 cu. in. per minute. The fact that the recorder readings are independent of the velocity of flow over such wide limits leads to a considerable simplification in the operation of the instrument.

The gas analysis cell is mounted in an oil bath, stirred by a small motor-driven propeller, and maintained at a constant temperature of 45°C by a heater controlled by a mercury thermostat. Fig. 7 shows the cell and its mounting, with the various compensation coils, to be described later.

The standard side of the cell (S, Fig. 4) was sealed with dry air in it.

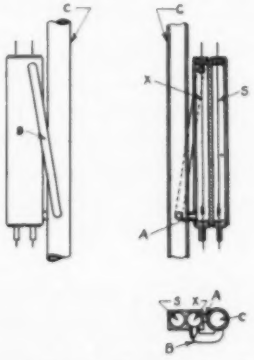


FIG. 4. *Gas analysis cell.*

CIRCUIT

The simple thermal conductivity bridge circuit is shown in Fig. 1. The actual circuit employed in the recorder is a modification of this circuit, as shown in Fig. 5. In this figure, S represents the slide wire

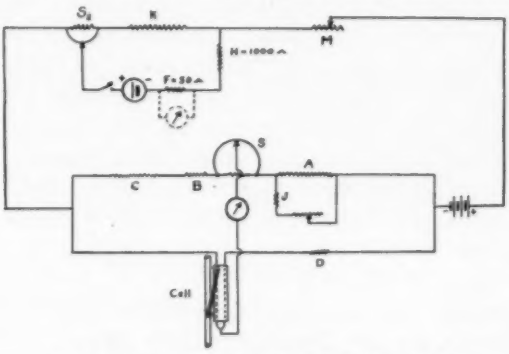


FIG. 5. *Actual thermal conductivity bridge circuit.*

mounted to be rotated by the recorder balancing mechanism; A and $B+C$ are the two extension coils of the slide wire, corresponding to A and B in Fig. 1.

D is a manganin coil of 0.08Ω, placed in series with the analyzing wire of the gas analysis cell. This coil serves as a current compensating device. That is, by the use of this coil, determined by experiment, the effect of variations in the bridge current on the balance point of the bridge is reduced to a minimum. It is evident that the two platinum wires S and X cannot be made identical in electrical characteristics and in their locations in the cells. Also, the thermal conductivity of gases varies with temperature, and the variation is different for different gases. Therefore, if on account of a change in the bridge current the platinum wire temperatures changed, the balance point of the bridge would alter. The use of the current compensating coil in series with one of the cell wires serves to render the temperature coefficient (apparent) of resistance of the two arms of the bridge including the cell wires the same, thus reducing errors due to bridge current variation. The normal bridge current is 0.265 ampere through the cell wires.

The coil B is a nickel coil of 0.228Ω, which acts as a temperature compensator. That is, if for any reason the cell temperature changed, the corresponding change in resistance of the nickel coil located close to the cell would tend to compensate the change in the balance point due to the change in cell temperature. This nickel coil was determined with a concentration of about 1.2 per cent CO_2 passing through the cell, this being the point of the scale at which the recorder was to be used most frequently. The compensating action evidently is only valid for one concentration of CO_2, but the errors due to temperature are considerably reduced over the whole scale range by its use.

The coil A is shunted by a coil J of 1000Ω and a resistance box variable from 1 to 9999Ω in steps of 1Ω. By this arrangement the resistance of the bridge arm A can be altered slightly, permitting the adjustment of the bridge so that the recorder will indicate zero per cent CO_2 when air is passed through the cell.

The rheostat M of 8Ω serves to adjust the bridge current to the proper value. The proper current value is obtained by balancing the potential drop across a resistance coil K, and a portion of a slide wire S_2 against the emf of a standard cell, using the recorder galvanometer as an index of the balanced condition. The recorder galvanometer is switched from its normal position in the gas analysis bridge to the standard cell circuit by means of a key.

A 6-volt storage battery with a trickle charger is used to operate the bridge. The recorder mechanism is operated from the 110 volt alternating current circuit.

GAS CIRCULATION

For the purpose for which this recorder was designed, it was desired to analyze the air from a greenhouse, which was supplied with carbon dioxide, keeping the concentration at about 1 per cent. The gas was drawn through sampling tubes located at different parts of the greenhouse to a central mixing chamber, and was then passed through two calcium chloride drying towers in series, then through a horizontal tube half filled with phosphorous pentoxide, and finally into the gas analysis cell. Great precautions are necessary in drying the gas, as small amounts of moisture cause considerable errors in the CO_2 readings. A water aspirator was used to obtain the suction.

CALIBRATION

The process of calibration, while slightly different for different gaseous mixtures, is in general the same in principle for all gases. The action of the temperature and current compensating coils can best be explained by a description of the method by which their values are determined. Also, the method of calibration is of interest from the standpoint of persons who might wish to make a definite calibration of a laboratory thermal conductivity gas analysis apparatus, utilizing the total slide wire length for a definite range of gas concentration.

The gas analysis cell, mounted in the oil bath at 45°C, was connected as shown in Fig. 6. *A* and *B* are resistance boxes variable in steps of 1Ω, while *D* is a resistance box variable in steps of 0.01Ω. *K* is a Kohl-

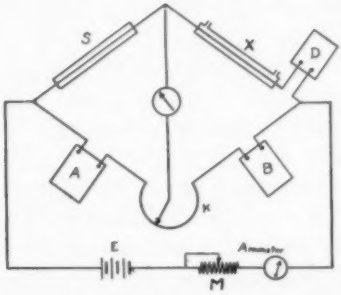


FIG. 6. *Connections for calibration of thermal conductivity cell.*

rausch slide wire with a resistance of 6.5349Ω, graduated with 1000 divisions. The boxes *A* and *B* were adjusted to approximately 1000Ω each, with box *D* at zero, and the bridge was balanced by adjustment of the slide wire *K*. During the above process the bridge current was

0.265 ampere, and a mixture of 1.2 per cent CO_2 and 98.8 per cent air was passed through the cell.

The first operation is that of determining the value of the current compensating coil *D*. Beginning with zero, the resistance in box *D* was increased in steps of 0.01Ω, and after each resistance change the effect on the balance point of the slide wire *K* caused by reducing the bridge current from 0.265 to 0.240 ampere was noted. A certain value, 0.08Ω, was found at which the balance point of the slide wire *K* was least affected by the change in the bridge current. A coil *D* of 0.08Ω was accordingly installed in series with the *X* platinum wire. All the end coils and compensating coils were mounted on a block placed close to the cell in the oil bath (Fig. 7).

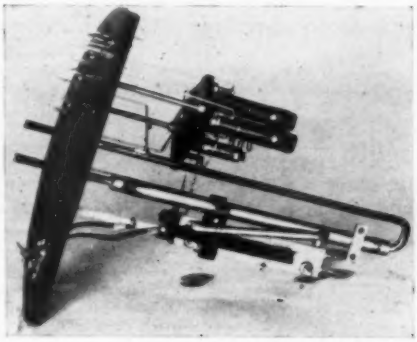


FIG. 7. *Gas analysis cell mounted, with coils.*

The next operation is that of determining the values of the end coils *A* and *B* for a given slide wire resistance. With dry air passing through the cell the following data were obtained:

Box A	*Box B*	*Slide Wire Balance Point, in Divisions*
912Ω	894Ω	535.2Ω

With the 1.2 per cent CO_2 mixture passing through the cell the results were as follows:

912Ω	894Ω	743.9Ω

The range of the instrument was to be 0 to 3.5 per cent CO_2, with the zero point one-eighth of the scale length from the lower end of the scale. This procedure was adopted in order to make the record easier to read when only a few hundredths of one per cent of CO_2 were being analyzed. Thus the actual range of the slide wire is to be 0 to 4 per cent CO_2.

Referring to Fig. 8, which represents the slide wire K with its end coils A and B:

m = balance point with air = 535.2 divisions
n = " " " 1.2% CO_2 = 743.9 divisions
p = " " " 4.0% CO_2

Slide wire range in divisions representing 1.2 per cent CO_2 = 743.9 − 535.2 = 208.7.

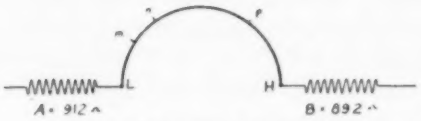


FIG. 8. *Slide wire and end coils.*

Slide wire range in divisions representing 4.0 per cent CO_2 = 4.0/1.2 × 208.7 = 695.7.

mp = slide wire resistance in ohms for the range of 4.0 per cent

$$CO^2 = \frac{695.7}{1000} \times 6.5349 = 4.543 \text{ ohms.}$$

A' = end coil resistance for low end of slide wire

$$= A + Lm = 912 + \frac{535.2}{1000} \times 6.5349 = 915.495$$

B' = end coil resistance for high end of slide wire
$= (A + LH + B) - (A' + mp) = 890.497.$

It was desired to use a slide wire in the recorder with a resistance of 0.125Ω. Therefore the above coil values were reduced in the ratio of 0.125 to 4.543. The actual end coil and slide wire resistance were then:

$mp = 0.125\Omega$
$A'' = 25.1897\Omega$
$B'' = 24.5019\Omega$

The next step was the determination of the value of the nickel temperature compensating coil. By allowing the oil bath to cool off slowly it was found that a change in temperature from 45°C to 20°C caused the balance point on the Kohlrausch slide wire to change from 785 divisions to 725 divisions, or 60 divisions. The equivalent change in resistance in ohms is 60/1000 × 6.5349 = 0.3920Ω, or 0.0157Ω per °C. To compensate this change in balance point a nickel coil must change by twice this amount, since the slide wire adds resistance to one arm

of the bridge and subtracts from the other. Thus the nickel coil must change by 0.0314Ω per °C. Reducing to the low value of end coils, using the ratio 0.125 to 4.543 as above, the nickel coil resistance change must be 0.000866Ω per °C. Taking the temperature coefficient of nickel at 45°C as 0.0038, the resistance of the nickel coil at 45°C is found to be 0.000866/0.0038 = 0.228Ω. Then since the value of the end A'' is 25.1897Ω, the coil C is 25.1897–0.228 or 24.9617Ω.

Referring again to Fig. 5, it will be noted that B'' as calculated is the resistance of the coil A shunted by the resistance of coil J and the resistance box in series. Assuming 1000Ω in the resistance box, the value of the coil A is found to be 24.805Ω.

Recapitulating, the coils as required for the bridge are as follows: (Fig. 5)

$S_1 = 0.125\Omega$
$A = 24.805\Omega$
$B = 0.228\Omega$ at 45°C (Nickel)
$C = 24.962\Omega$

OPERATION

This apparatus has been in operation for some months, analyzing air from a greenhouse used for researches in plant growth. The greenhouse is supplied with carbon dioxide from outside sources so that the concentration is usually between one and two per cent. Tests indicate that the instrument is accurate to about 0.05 per cent of CO_2.

The slide wire and its connecting leads are the only part of the bridge circuit which are not contained in the oil bath. The connections from the cell to the oil bath are made in such a way that the lead resistances are added to the slide wire end coils A and B, rather than to the fine platinum wires S and X. In this way the effect of variation of lead resistance is minimized.

The whole apparatus is automatic and self-recording. The only attention necessary is to regulate the bridge current at intervals, depending on the constancy of the battery voltage. The drying agents need to be renewed occasionally, depending on the relative amount of moisture contained in the gas.

A representative record of this apparatus is shown in Fig. 9. The range is, as stated before, 0 to 3½ per cent CO_2, the zero of the scale being 1/8 of the scale length from the left end of the scale. Each small division of the paper represents 0.04 per cent of CO_2. The recorder itself is accurate in its self-balancing action to 0.02 per cent of CO_2.

Similar apparatus can be constructed for the analysis of other gases, where greater sensitivity is demanded than is available using the ordinary thermal conductivity methods. The use of the current and temperature compensation coils makes the attainment of higher

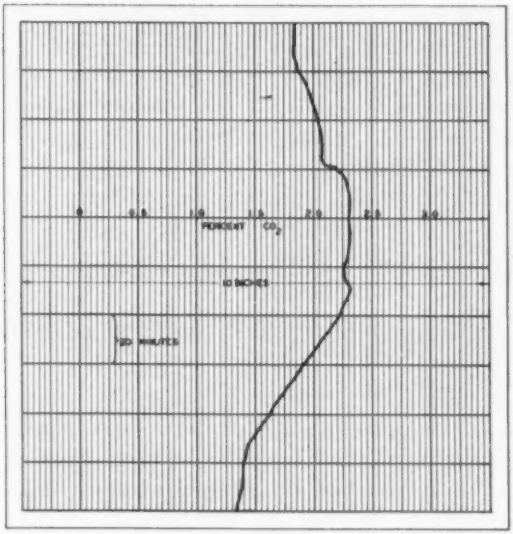


FIG. 9. *Specimen chart from recorder.*

accuracy possible, while the addition of the cell construction shown in Fig. 4 renders the instrument much easier to operate as a continuous recorder, since no close regulation of the velocity of flow of the gas sample is required.

Mr. J. C. Peters, Jr., was largely responsible for the design of this apparatus, and much credit is due to him for his work.

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AN EXPERIMENTAL STUDY OF THE PISTON PRESSURE GAGE TO SIX HUNDRED ATMOSPHERES*

By Frederick G. Keyes and Jane Dewey

INTRODUCTION

The "dead weight" piston gage has come into extended use during the past twenty years as an accurate pressure measuring instrument. The gage consists essentially of a uniform cylindrical piston fitting suitably into a cylinder and supporting, at the exposed end, a known weight which balances the hydrostatic pressure exerted by a pressure transmitting fluid on the lower surface of the piston.

It is evident that the weight required to equilibrate a given hydrostatic pressure will be a function of the area of the piston. For a piston closely fitting its cylinder, the weight required is assumed to be proportional to the mean area of piston and cylinder. In practice, however, it is a difficult task to determine the area or rather diameter of the cylinder with sufficient accuracy and recourse can be made to calibration of the gage against a pressure standard. The latter ultimately must of course be interpreted in terms of the pressure exerted by a column of mercury. In this laboratory the mercury-in-glass column is 900 cm in length while prior to 1916 a column 1280 cm long was available. All pressure measurements which have hitherto been made in this laboratory are referred directly to a calibration of the gage used, against a mercury column and the gage "constant" thereby determined has been employed to extrapolate to pressures as high as 2000 atm.

The uncertainty which may attend this long extrapolation has been a matter of concern and plans have from time to time been discussed in detail for erecting a column in a suitable mine shaft. The expense, however, of carrying out the project is so considerable that each year has seen the project postponed. The decision has been made therefore to employ in principle the method used by Holborn and Schultze[1] whereby use is made of a column of moderate length with two gages having pistons of substantially equal diameter.

The Holborn and Schultze apparatus consists of a column of about 1700 cm in length, at the upper end of which a gage is placed while a second and similar gage is installed at the lower end of the column.

* Contribution from the Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology, No. 179.

[1] Holborn and Schultze, Ann. d. Phys., *47*, p. 1095; 1915.

After each gage had been compared with the open column, the pressure at the upper gage may be adjusted to equal one column length, thereby producing a pressure of two column lengths at the lower gage. By continued interchanges of the gages, comparisons of the gage "constants" may be made to any desired pressure within the limits of pressure for which the columns and gages are designed to operate.

The foregoing principle has been applied to obtain the measurements herein recorded. In our application certain modifications in the details of the disposition of the apparatus have been employed, such for example as installing both test gages at the lower level of the column and making the exchange of gages by suitable manipulation of stopcocks rather than physically interchanging the top and lower gages.

A mercury-in-glass column of 900 cm length was first employed to obtain the low pressure gage "constants." Each gage was subsequently used to determine the length of a mercury column in a steel tube of very nearly the same length as the mercury-in-glass column. The steel column was water jacketed and pressure transmission from the upper end of the column to the second gage was secured by means of oil contained in a tube inclosed within the same water jacket as the mercury column.

The method of detecting pressure equilibrium[2,3] was essentially the same as that in use in this laboratory for pressure measurements for the past fifteen years. This electrical method enabled the pressure comparisons to be made with speed and great accuracy.

Some years ago Professor Beattie made some comparisons of a piston gage with the 900 cm mercury-in-glass column employing oils of different viscosity. The "constant" obtained was independent of the oil viscosity within the limits of error of the comparison which was about 1 part in 20,000, thus confirming what would be inferred on theoretical grounds.[4] The pressure range, however, was small (12atm.)

[2] Keyes and Brownlee, Thermodynamic Properties of Ammonia, Wiley & Sons, p. 11; 1916.

[3] Keyes, Joubert and Smith, Jour. Math. & Phys., M.I.T., p. 195; 1922.

[4] For the steady motion of a cylindrical shell of incompressible fluid moving upward in the direction of the z axis the usual hydrodynamical equation is:

$$\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = \frac{\partial p}{\partial z} = \text{constant}$$

where μ is the coefficient of viscosity, w the velocity along the direction of the z axis, p the pressure per unit area, and r the radius of the cylinder. The space between the piston and cylinder for the case of exact coincidence of the two axes may be designated $(a-b)=\sigma$ where

and it might be desirable to extend this investigation to higher pressures using the present calibration system. It should be emphasized that the theory of the effect of the escaping oil indicates independence of the gage indications from viscosity variations for an incompressible oil. In the present paper the same oil was used throughout the measurements.

The ideal piston and cylinder, namely one absolutely rigid and inexpansible with heat, would disclose a "constant" on comparison with the mercury column which would be independent of the pressure. The gages employed in the present work consist of a hardened piston and cylinder, the former being ground and lapped to fit the lapped hole of the cylinder. The primary purpose of this investigation was to determine by experiment the effect of the stress system set up in the piston and cylinder on the effective area under the usual conditions of use. The elasticity theory has already been envoked to indicate the nature of the effect which distortion of the piston and cylinder may have upon the effective area of the piston. Thus the effective area of the 0.159 cm piston used by Bridgman[5] was calculated to change due to the stress system by $1.8\times10^{-7}\times p$ kgm. The effect would require a correction of about 1/500 in the pressure at 10000 kgm.

Michels[6,7] has recently elaborated in detail the theory of the Schäffer and Budenberg differential piston gage based on the usual elasticity theory and made some measurements of the effective area of a differential piston of the Schäffer and Budenberg type. The latter measure-

a and b are the radii of cylinder and piston respectively. If no "slip" occurs at the solid boundaries $\partial w/\partial r$ is zero as usually assumed. Integration of the equation above gives

$$\mu\frac{\partial w}{\partial r} = \frac{\sigma}{2}\frac{\partial p}{\partial z}\frac{a+b}{r}$$

Since a is nearly equal to b, $(a+b)/r$ may be taken as very nearly 2 and the total force on the cylindrical surface perpendicular to r becomes:

$$F = \pi a\sigma\frac{\partial p}{\partial z}l = \pi a\sigma(p_1 - p_0)$$

This shows that the force due to the escaping oil and tending to lift the piston is independent of the viscosity of the oil and remains an invariable fraction of the pressure measured. The equation is in agreement with Michel's which was obtained by a different method involving the same assumptions.

If compressibility of the liquid is allowed for, an equation of state of the liquid must be assumed. The result obtained above, however, would be but slightly modified.

[5] P. W. Bridgman, Proc. Am. Acad., *47*, p. 321; 1911.

[6] A. Michels, Ann. der Phys., *72*, p. 285; 1923.

[7] A. Michels, Ann. der Phys., *73*, p. 577; 1924.

ments made use of a closed hydrogen manometer provided with electrical contacts sealed into a hydrogen closed manometer tube whereby the same setting could be very exactly reproduced. By balancing the pressure corresponding to the hydrogen manometer setting first against the gage and second against the gage plus a 500 cm mercury column, it was possible to compute the gage constant at the pressures corresponding to the fixed contacts of the hydrogen manometer. Measurements were carried out in this manner from about 40 atm. to 178 atm. There was no detectable trend in the effective areas measured. The consistency of the observations on the other hand was about 1 in 3000. Application of the theory to the particular piston used by Michels indicated that at the highest pressure actually measured (178 atm.) the correction for distortion would be much less than 1 in 3000.

DESCRIPTION OF APPARATUS

The gages used in this investigation are similar in design to those previously in use in this laboratory with the exception of some improvements in the design of the piston oscillating mechanism and the design of the piston-cylinder. The former produces a uniform motion in both directions whereas formerly the motion was non-uniform. The piston-cylinder design is such as to use cylinders (of fixed external dimensions) carrying different diameter pistons and thus the five standard sizes of piston in use for general pressure measurements can be employed in the same piston-cylinder block. The range of pressures which can be measured extends from several atm. to about 2000 atm.

In the reproduction of the photograph Fig. 1, there is shown the motor, gage block and gage complete except for the scale pan. The piston and cylinder is not visible being hidden by a brass cylindrical support surrounding the nut which holds the cylinder into the block. Attached to the upper portion of the piston is an arm extending at right angles which is moved back and forth through an angle of 30° by means of a fork, bearing hardened rollers and oscillated by the motor. The oscillation rate is 30 per minute. The cross beam from which the scale pan is provided with a ball bearing at the point where it is supported on the piston.

The effect of temperature on the effective diameter requires attention in the present work since a variation of 1° produces a change in effective piston diameter in amount 2.4×10^{-5} or 1 part in 42000. The variations in operating temperatures of the gages are large enough to make the

application of this correction necessary in view of the reproducibility of the comparisons which will be presented.

Two types of scale pan are used with the gages, the larger of which weighs somewhat less than 20 kilos and is used with a total load of 300 kilos. The lighter pan of aluminum is used for the measurements

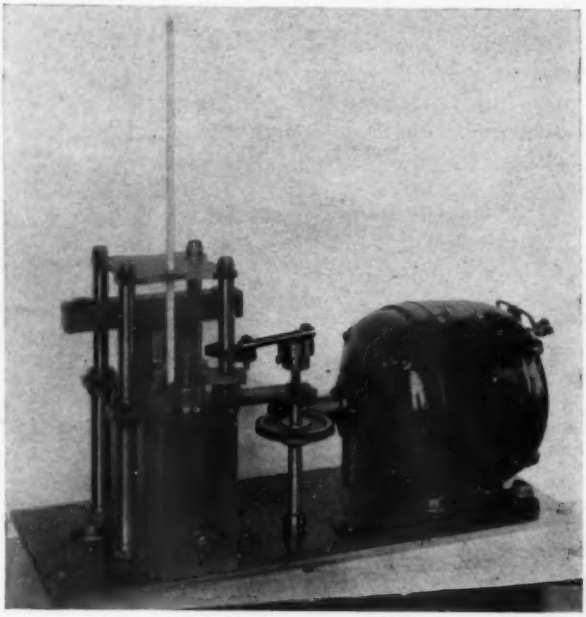

FIG. 1. *Pressure gage.*

of pressure to 20 atm. and is employed in calibrating the gage against the mercury in glass column as well as in the present work for ascertaining the length of the mercury in steel column. Fig. 2 shows the disposition of the gates with scale pans, oil injector, electrical contact, detecting apparatus, and stopcocks used for interchanging the roll of the gages from top to bottom of the mercury in steel column.

Two sizes of piston were used in the present investigation referred to as No. 8 and No. 9 large, and No. 8 and No. 9 small. The former had an area of about four fold the latter. Each of these pistons was calibrated against the mercury in glass column. The constants of the

gages (reciprocal of effective area divided by 1/10th the density of mercury) will be expressed as the number of grams weight which enables

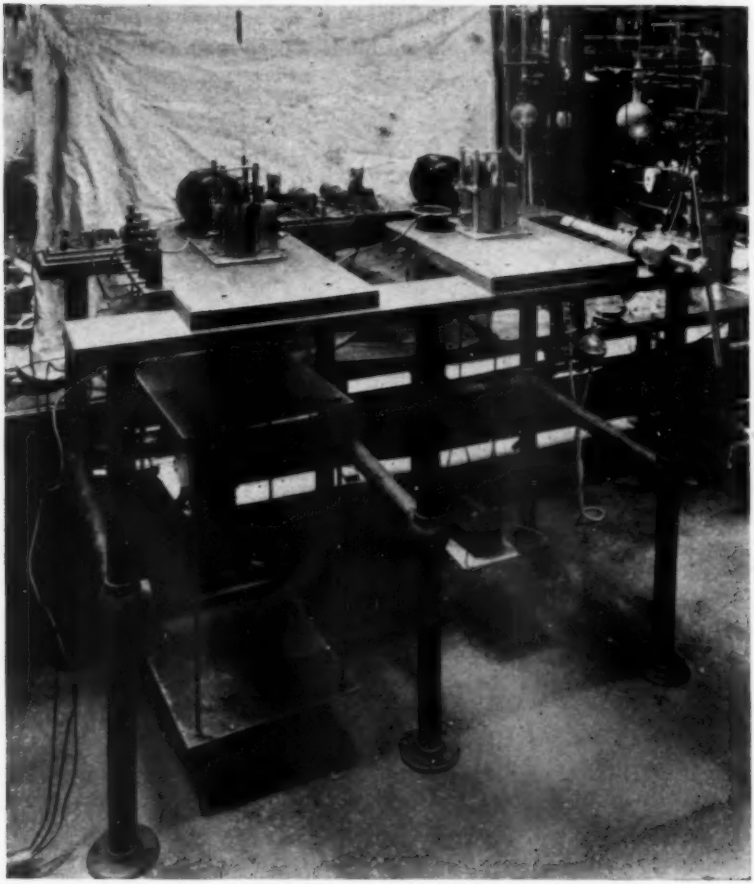

FIG. 2. *Actual arrangement of gages and stopcocks.*

the piston at 30° to support 1 mm of mercury at the temperature of melting ice and corresponding to a gravitational constant of 980.397.*

* This is the value of the constant at this laboratory. Since in this paper relative values are alone in question, reduction was not made to the international value used for pressures (980.665).

THE MERCURY-IN-GLASS COLUMN

The glass column 900 cm in length was contained and supported in a vertical rectangular shaped trough, the front of which could be covered with card board covers to assist in maintaining the temperature constant. Thermometers were placed in proximity to the column and enabled the mean temperature of the column to be ascertained. The column could be filled to any desired height by injecting oil by means of a Cailletet pump into a closed vessel containing mercury from which a tube lead to the lower end of the glass mercury column. The gages could be connected by means of an oil lead to the short arm of the manometer, which was provided with a platinum contact. The disposition of apparatus was in fact similar in every respect to that described and illustrated some years ago[8] with the exception that the electrical contact equilibrium indicator was sealed directly into the glass of the manometer short arm.

The height of the column was measured by a standard steel tape stretched under a load of 6 kilos and calibrated against a meter bar which in turn had been compared at the United States Bureau of Standards. All large weights used had been compared with the Massachusetts State House Standards[9] while the smaller set was compared with a one kilo set of weights calibrated at the United States Bureau of Standards. All results of the column comparisons are reduced to gage temperature 30° and the results presented in table 1.

TABLE 1. *Gage constants at 30° from comparisons made with a mercury column 900 cm in length.*

	Size	Gage constant
Piston number 8	large	0.499370 mm per g. $\pm 0.15 \times 10^{-4}$
" " 9	"	0.499405 mm per g. $\pm 0.24 \times 10^{-4}$
" " 8	small	1.99637 mm per g. $\pm 1.0 \times 10^{-4}$
" " 9	"	1.99479 mm per g. $\pm 1.4 \times 10^{-4}$

It will be noted that the average deviation from the mean for No. 8 and No. 9 large piston is about 1 part in 25000 and that the effective areas are within 1 in 14000 of one another. The smaller piston No. 8 and No. 9 are not so closely of the same effective area.

[8] The Thermodynamic Properties of Ammonia. John Wiley & Sons, p. 11, 1916.

[9] We take pleasure in expressing our great obligation for the painstaking aid given us and the courtesy shown by Mr. Francis Meredith, Director of Standards of the State of Massachusetts.

THE MERCURY IN STEEL COLUMN

To withstand the pressures contemplated in the comparison of the gage indications at higher pressures it was necessary to employ a mercury-in-steel column. The details of construction are represented in Fig. 3. The length of the column was about the same (875 cm as compared with 900 cm) as the glass column. The bore of the column was 1 cm with a wall thickness of 0.476 cm. In the figure the steel column is designated A and the end is threaded and formed to make the connecting joint directly on the tube. The block B receives the latter through which it communicates to the manometer short arm; a Cailletet pump used to inject oil which forces mercury into the column, and a piston mercury injector, I, for fine adjustment of column height. Each end of the column is provided with a mercury-oil contact needle R_1 and R_2 which carries an oil connection leading to the stopcocks. Connection is finally made from the latter to the gages.

The electrically insulated mercury-oil contact needles are connected with an electrical device for giving visual indication when contact is made or broken at the mercury-oil junction. The electrical apparatus consists of a relay operating on 0.002 amp. used to connect the circuit to a four volt flash light bulb as illustrated in the drawing. With the device sketched a very minute current passes at the needle mercury miniscus. The light comes on promptly when the contact is broken and extinguishes when contact is made. This electrical circuit is the result of a sustained effort to improve the contacting device and is very satisfactory. It has been found essential to have as small a current as possible pass between the needle and mercury otherwise emulsification of the mercury in the oil takes place disturbing the distinctness and reproducibility of the contact.

The drawing aims to make clear that the manipulation of the stopcocks S_1, S_2, S_3, S_4, permits either gage to be connected to the top or bottom of the column at will.

The top of the column is connected to the gage by means of the tube C of 0.7 cm internal diameter. The oil used was a clear grit free "vacuum" oil whose density was found to be $0.9083+0.000534t$ and compressibility 53.1×10^{-6} cc per atm. per cc at 25°.

The two gages G_8 and G_9 are shown in end view together with the oil injector connected to one of the gages. Each gage was provided with an oil injector in practice in order that oil could be supplied to make up for leak and to displace the pistons as required.

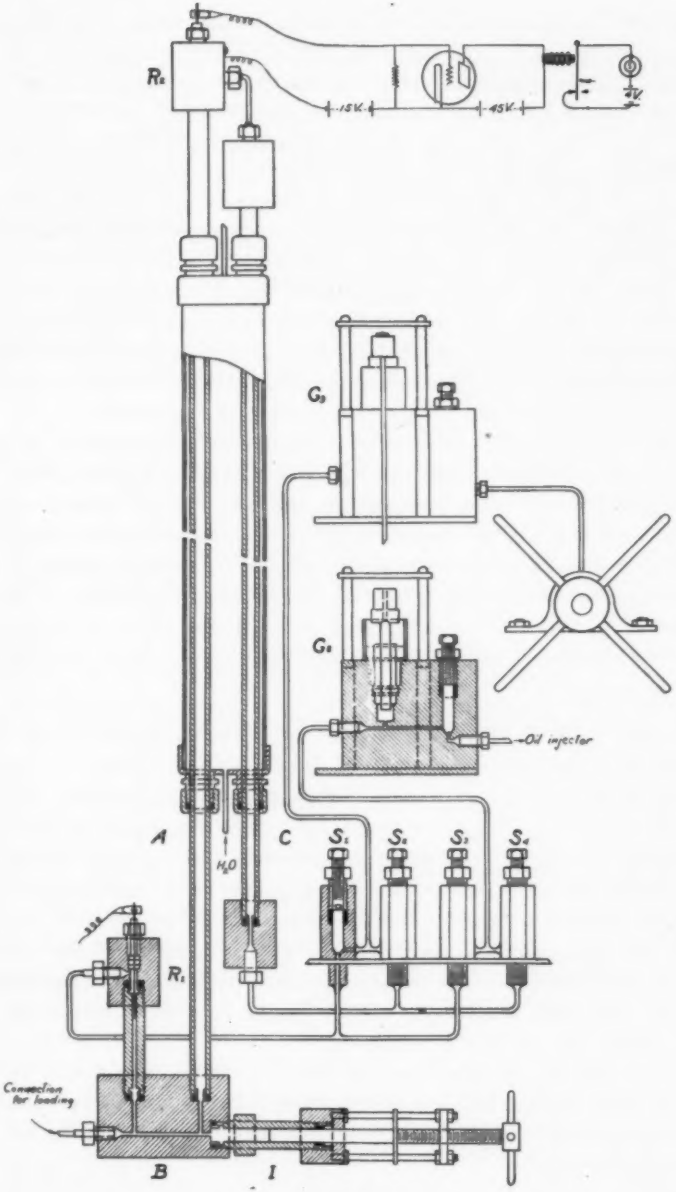
R₂
15V.
45V.
4V.
G₂
G₁
Oil injector
A
C
H₂O
S₁
S₂
S₃
S₄
R₁
Connection for loading
B
I

FIG. 3.

Both the mercury-steel and oil columns were inclosed in the same brass water jacket consisting of a 7.5 cm brass tube lagged externally with a standard pipe covering. It was found preferable to circulate water in a closed circuit by means of a gear pump rather than allow the water from the mains to flow through the jacket. The latter procedure produced a temperature difference between top and bottom of the column of from 2 to 4 degrees whereas the closed circuit arrangement lead to a difference never greater than one degree. The actual temperatures along the column were read by means of three thermometers placed at the bottom, middle and top. The bulbs of these thermometers rested between the mercury tube and the oil tube. The top of the column was exposed for a short distance but it was lagged with hair felt and the temperature obtained by means of two thermometers placed along the exposed portion of the column.

The loading of the column with mercury was accomplished by first exhausting the column and subsequently admitting mercury from the mercury reservoir and pumping in the mercury by means of the Cailletet pump. When approximately the correct amount of mercury had been introduced additional mercury was injected by means of the mercury injector *I*. By means of the latter the mercury could be adjusted to very exact contact with the insulated needles at top and bottom and thus a fixed length of mercury could be made available at all times.

CORRECTIONS APPLIED TO THE MERCURY IN STEEL COLUMN

The temperature dilation of the pure mercury and its compressibility must be accurately known to reduce the column lengths of mercury in the steel column to the force per unit area which corresponds to the defined unit of pressure. Reduction was not made to the international pressure unit with respect to gravity since relative pressure values suffice in the present investigation.

In addition to the mercury constants there is required the stretch of the steel column under internal pressure and also the expansivity of the steel with temperature. The constants used in reducing the observations are contained in Table 2.

The length of the steel column between needle points was determined under a slight pressure (1 atm.) exerted at the top of the column. This length as determined by using both the larger pistons was 875.12 cm ± 0.09 at 22.8°.

TABLE 2. *Properties of mercury, steel and oil used in reducing column lengths of mercury to standard conditions.*

Mean volume expansion of mercury (0–25°)	1.82×10^{-4} cc per cc
Mean linear expansion of steel (0–25°)	0.1184×10^{-4} cm per cm
Compressibility of mercury 22°	$(3.94-1.1\times10^{-4}\ p\text{ atm.})\times10^{-6}$ cc per cc
Young's modulus for steel 20°	2.2×10^{6} atm. per $\overline{\text{cm}}^2$
Area of steel column section	2.136 cm^2; bore of column 0.712 $\overline{\text{cm}}^2$
Density of oil used	$(0.9083-0.00053\ t)$ gm per cc
Compressibility of oil used	53.1×10^{-6} cc per cc per atm. at 25°

EXPERIMENTAL RESULTS

The data obtained have been reduced and listed in Table 3 for the larger pistons while the differences between true and indicates pressures are plotted in Fig. 4, for the smaller pistons. The method of obtaining

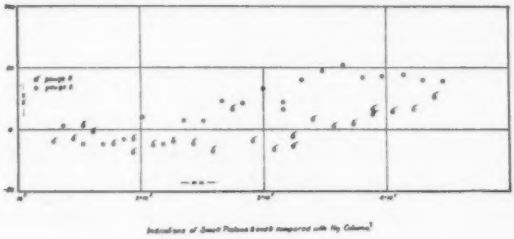

FIG. 4.

the pressures in column 9, each entry of which corresponds to a certain number of column lengths is as follows. The first entry of column 3 (Table 3) is the pressure exerted by the gage connected to the upper end of the column. This pressure is computed from the total weight applied to the piston using the gage constant determined from the comparisons made with the gage against the mercury-in-glass column. The pressure of the single column lengths appear in column 5. To 1474.7 mm there must be added then 8137.6 mm giving 9612.3 mm (9th column) which is to be compared with the pressure measured by the gage connected to the lower end of the column. The gages are next interchanged without altering the weight applied to the piston which in the former setting was connected to the lower end of the column. Equilibrium is again obtained, the lower gage now indicating a pressure of 17748.2 mm. The mercury column pressure with which this is to be compared is 9612.3 mm (column 9, 1st row) plus an additional length of mercury 8137.5 mm corrected for any difference in pressure exerted by the upper gage as a consequence of a temperature change in

the latter from the time it was used to measure a pressure to the time it was used to apply a pressure at the top of the column. This difference is obtainable as the difference between columns 8 and 3 for the same gage. For example the sixth entry shows a pressure of 25890.3 mm exerted at the top of the column by gage 8 at 30° which in entry 5, 8th column, was at a temperature of 27° the difference of 1.7 mm corresponds to the pressure difference arising from the increased area of piston 8 due to 3° rise in temperature. The numbers in column 9 are therefore pressures obtained by successively adding column lengths of mercury to the small initial pressure of 1474.7 mm and the 8th column represents the pressures computed from the gages using the constants obtained in the low pressure comparisons with the glass column. From Table 3 on comparing columns 8 and 9 it will be evident that no particular trend is disclosed and to 160 atm. the effective piston area is not sensibly altered.

Fig. 4 exhibits the reduced data pertaining to the smaller diameter pistons. The initial pressure used corresponds to the limiting pressure to which comparisons were carried in the use of the large pistons. The limit of pressure attained in the comparisons of the small pistons with the mercury column is about 587 atm.

The indication is that a small trend exists in the pressures given by the small pistons of which piston 9 shows the larger trend. The data plotted in Fig. 4 indicates clearly that at the higher pressure range the indications of gage 9 are on average higher than those of gage 8. The mean deviation is however but one part in 10,000 greater than the mercury column pressure.

Without doubt the difference in the indications of the two gages is to be traced to differences in the fit of the pistons in their respective cylinders, although the precise nature of the causes underlying the difference is not known at present. It has already been noted that a piston which fits its cylinder too closely is not reliable in its indications. A test of piston suitability which has been attempted in a preliminary manner is to apply an electric current to the piston and cylinder. It appears that a reliable piston-cylinder always gives perfect insulation of one member from the other during the oscillation, contact being made of course at the reversal of the oscillation. The inference from this fact would be that the piston must be actually "floating" on the oil during the major portion of the time that it is acting as a pressure measuring device.

It is now, so far as known, generally admitted that oscillation of the piston is to be preferred to complete rotation; a conviction of one of the present writers some fifteen years ago. It is now believed, as a consequence of observing the insulation characteristics of various

TABLE 3. *Mercury column comparisons made with large pistons.*

Gage connected to upper end of column			Mercury column	Gage connected to lower end of column			Mercury column	
#	T °C	Pressure mm	T °C	Pressure mm	#	T °C	Pressure mm	Total pressure mm	Deviation mm
9	29	1474.7	23.4	8137.6	8	30	9611.5	9612.3	−0.8
8	30	9611.5	23.5	8137.5	9	29	17748.2	17749.8	−1.6
8	27	9612.1	19.7	8142.1	9	26	17752.3	17755.0	−2.7
8	28	9611.9	20.7	8140.9	9	27	17752.3	17753.6	−1.3
9	28	17751.9	20.9	8140.7	8	27	25892.0	25893.9	−1.9
8	30	25890.3	22.7	8138.5	9	29	34029.1	34039.7	−1.6
8	30	25890.3	22.3	8139.0	9	30	34030.9	34031.2	−0.3
9	31	34030.3	26.1	8134.6	8	31	42162.7	42165.2	−2.5
8	31	42162.7	26.5	8134.0	9	31	50300.5	50299.2	+1.3
9	31	50300.5	27.1	8133.5	8	29	58428.8	58432.7	−3.9
8	27	58431.1	25.2	8135.7	9	31	66577.5	66570.7	+6.8
8	26	58432.3	19.5	8142.5	9	26	66585.9	66578.7	+7.2
9	30	66579.3	20.3	8141.7	8	27	74711.4	74713.6	−2.2
8	29	74708.4	20.8	8141.1	9	30	82853.8	82851.7	+2.1
9	27	50304.6	21.6	8140.0	8	27	58435.9	58443.3	−7.4
8	29	58428.8	20.8	8141.0	9	30	66575.1	66577.2	−2.1
9	28	66577.8	21.5	8140.2	8	28	74713.8	74720.1	−6.3
8	29	74712.3	21.6	8140.1	9	29	82861.0	82858.7	+2.3
9	30	82859.3	21.7	8140.1	8	30	90994.6	90997.1	−2.5
8	23	91016.9	20.0	8142.2	9	23	99157.8	99161.6	−3.8
9	26	99153.9	20.7	8141.4	8	25	107297.0	107299.1	−2.1
8	29	107288.3	21.9	8140.1	9	27	115428.0	115430.5	−2.5
9	22	115439.5	22.0	8140.0	8	22	123583.9	123582.0	+1.9

piston-cylinders under actual working conditions, that the reversal of motion serves better to maintain the piston more nearly in a mean central position within the cylinder. Much carefully planned experimental work is evidently needed to complete our knowledge of this very satisfactory pressure measuring device.

SUMMARY

(1) The method of extending comparisons of the piston type pressure gage with a mercury column used by Holborn and Schulze has been

modified. The modifications consist of (1) water jacketing the column to secure constancy of temperature, (2) providing fixed electrical contacts at both ends of the column which with the aid of a sensitive device for injecting or withdrawing mercury provides an invariable column length of mercury, (3) an arrangement of steel stopcocks and water jacketed oil line from the top of the column whereby both gages may be used at the floor level.

(2) Series of comparisons have been made of two pairs of piston gages of different effective areas with a mercury column (875 cm long) in a water-jacketed steel tube to 160 atm. for the larger piston and to 587 atm. for the smaller piston.

(3) The larger piston indicates true pressures to the limit of pressure for which it was designed; 160 atm. The average deviation of the true and indicated pressures is one part in 40,000.

(4) The smaller piston indicates slightly higher pressures than the true pressure beginning at about 400 atm. The deviation increases to one part in 11,000 at 587 atm.

(5) The causes of the discrepancy at the higher range of pressure for the smaller piston cannot be adequately discussed until further investigations have been completed. These investigations are concerned with the precise effects dependent on closeness of fit of the piston in its cylinder, and the changing conditions of oil flow along the piston at higher pressures as well as the distortion of the cylinder due to the mounting and pressure effects on both cylinder and piston.

To interpret length of mercury column as pressure, the compressibility of mercury as a function of the pressure should be accurately verified and also the dependence of temperature expansion on pressure.

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THE OPTICAL SYSTEM OF THE OSCILLOGRAPH AND SIMILAR RECORDING INSTRUMENTS

By Arthur C. Hardy

There is scarcely a field of investigation today which does not at some time employ a strip of photographic film or paper to record the movement of a spot of light. A typical example is the galvanometer or oscillograph mirror which, by its rotation, produces a displacement of the spot and thus leaves a developable image on a strip of moving film. Instruments of this sort are used so frequently that a lengthy discussion of the various methods of application is unnecessary. Suffice it to say that mechanical considerations involving the moment of inertia and natural period of the mirror often require that the latter be small. By reason of its small size, there is difficulty in obtaining sufficient illumination on the film, especially when the velocity of the spot on the film is rapid. Moreover, unless the optical and photometric requirements of the system are accorded due consideration, the size of the spot may be so large as to reduce the precision with which the record may be read. It is the purpose of the present paper to discuss the photometry of optical systems of this sort and to derive expressions for the illumination on the film and the size of the spot. These results should be of use to the designer and the user of instruments of this class and for some reason they seem to have been unavailable heretofore.

It is convenient to divide the optical systems to be considered into two general classes. In the first class, a centered system consisting entirely of spherical lenses is assumed, while cylindrical lenses are used in systems of the second class.

CLASS I. THE OPTICAL SYSTEM CONSISTS OF A CENTERED SYSTEM OF SPHERICAL LENSES

A typical system of the first class is represented in Fig. 1. A single lens is shown between the source and the mirror and another between the mirror and the film, the constants of the pair being so chosen that an image of the source is finally focussed on the film.[1] Although only single lenses are indicated, any combination of centered spherical lenses is possible, the only change being that the values of u and v must then

[1] It is possible to obtain slightly greater film illumination on the axis of the optical system by displacing the film a short distance toward the exit pupil. However, the illumination of the film is then not uniform and the actual gain is small.

be measured from the corresponding principal planes. For simplicity the mirror has been drawn as though it produced no deflection of the beam and operated only as a stop in the optical system. The results derived will be strictly applicable only when the rays fall normally on the mirror surface. If they fall at any other angle, the result is to reduce the effective width of the mirror (and therefore the illumination on the

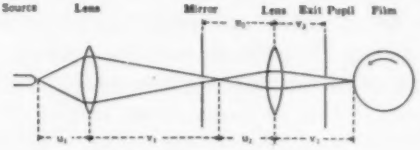

FIG. 1.

film) in proportion to the cosine of the angle of incidence. This factor has been omitted in what follows, since in most practical applications, the light does fall nearly perpendicularly to the mirror surface.

In any optical system of this class, there is some one physical stop that limits the extent of the axial zone pencils, known as the *aperture stop* of the system. In general, it is the rim of one of the lenses or some diaphragm inserted for the purpose. In this discussion we may assume that it is possible to obtain lenses of unlimited aperture so that the mirror is ordinarily the aperture stop of the system as shown in Fig. 1, since it must be small because of mechanical considerations. The image of the mirror formed by the lens or lenses preceding it is the *entrance pupil* of the system and the image of the mirror formed by the lenses following it is the *exit pupil*.

Neglecting the loss of light in the system due to absorption, reflection, and scatter (which will amount to 50 per cent or more in such a system as this), the illumination on the film (lumens per cm²) is the product of the brightness of the source (candles per cm²) and the solid angle subtended by the exit pupil at the film (for small solid angles). Thus, if we assume a rectangular mirror of width w_3 and length l_3, the dimensions of the exit pupil, w_4 and l_4 are given by the following expressions. (See Fig. 1.)

$$w_4 = w_3 \cdot \frac{v_3}{u_3}$$

$$l_4 = l_3 \cdot \frac{v_3}{u_3}$$

With sufficient accuracy, the illumination on the film (I) is given in terms of the brightness (B) of the source by

$$I = B \cdot \frac{w_4 l_4}{(v_2 - v_3)^2} = B \cdot \frac{v_3^2}{u_3^2} \cdot \frac{w_3 l_3}{(v_2 - v_3)^2} \tag{1}$$

It will appear at once that the illumination becomes very intense as $(v_2 - v_3)$ approaches zero. In the limiting case, the source would be focussed on the mirror and the mirror on the film. The duty of limiting the zone pencils would then be taken over by one of the lenses of the system and by making the lenses of sufficient aperture, the illumination could be made very intense. However, this arrangement is undesirable since no lateral displacement of the spot of light could result even from large deflections of the mirror.[2] The merit of any particular optical system should be judged therefore only after a consideration of the amount of deflection of the spot produced by a definite rotation of the mirror.

Suppose the mirror to rotate through an angle α. The deflection D of the spot on the film is then given by

$$D = 2\alpha(u_3 - u_2) \cdot \frac{v_2}{u_2} = 2\alpha v_2 \left(\frac{u_3}{u_2} - 1\right) = \frac{2\alpha}{\frac{v_3}{u_3}\left(\frac{1}{v_2 - v_3}\right)} \tag{2}$$

remembering that

$$\frac{1}{u_2} + \frac{1}{v_2} = \frac{1}{u_3} + \frac{1}{v_3}$$

For a constant value of the ratio D/α, the denominator of the last member of equation (2) is constant. This quantity appears squared in the last member of equations (1). *The illumination is therefore independent of the particular lens system used between the mirror and the film.* In other words, the focal length of this lens might as well be infinite, in which case it could be omitted altogether without altering the illumination on the film. Consequently, *the illumination produced by a single lens is equal to that obtainable by any spherical lens system whatever.* The single lens may be placed anywhere between the source and the film without affecting the illumination, provided that it forms an

[2] If the system is not free from spherical aberration, there will be a small deflection of the spot as the mirror rotates. However, with a perfect image forming system which we shall assume, there can be no displacement of the spot when the mirror is imaged on the film.

image of the source on the film and is of sufficient diameter to avoid being the aperture stop of the system.[3]

The best position for the lens depends somewhat on the nature of the instrument. With the ordinary galvanometer, a very satisfactory arrangement is to use a concave spherical mirror of such radius of curvature that an image of a diaphragm placed close to the source is focussed on the film.[4]. Let us assume a system consisting simply of a source, a diaphragm (field stop), a concave spherical mirror and a rotating film. If the width and length of the diaphragm are given by w_2 and l_2 respectively, the dimensions of the spot on the film are

$$w_5 = w_2 v/u \tag{3}$$

$$l_5 = l_2 v/u \tag{4}$$

where u is the distance of the diaphragm from the mirror and v is the distance of the mirror from the film. A rectangular spot has been assumed, since a spot of this shape is always obtained in instruments of the second class and its assumption here facilitates future comparison of the two systems. In case a circular spot is desired in systems of the first class, either w_5 or l_5 in equations 3 or 4 may represent its diameter. The chief advantage of a circular spot is that the width of the line traced on the film is then nearly independent of its slope. A rectangular mirror has been assumed since a mirror of this form may have a lower moment of inertia for the same area.

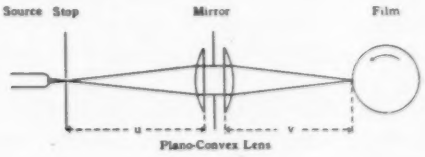

FIG. 2.

If the mirror is to be damped by placing it in oil, as is done in the oscillograph, a more satisfactory arrangement is to use a plano-convex lens as shown in Fig. 2. The lens takes the place of the usual window and reduces the number of air-glass surfaces in the system, each of

[3] Since the more logical position of the lens is between the source and the mirror, this is equivalent to making the angle subtended at the film by the lens greater than that subtended by the mirror.

[4] The diaphragm opening may then be considered as the source and the illumination remains unaltered if the actual source is of sufficient size. It must, of course, be large enough to avoid being either a field or an aperture stop of the system.

which causes a loss of light by reflection of approximately 5 per cent. Since light is twice transmitted by the lens, its focal length is obtained from

$$\frac{2}{f} = \frac{1}{u} + \frac{1}{v} \tag{5}$$

This, of course neglects the distance between the lens and the mirror in comparison with other dimensions of the figure. When this assumption is not possible, resort may be had to the more general methods employed in the handling of thick-lens problems.

The illumination with such a system as shown in Fig. 2 is simply

$$I = B \cdot \frac{w_3 l_3}{v^2} \tag{6}$$

The deflection of the spot produced by a mirror rotation α is

$$D = 2 \cdot \alpha \cdot v \tag{7}$$

The size of the spot on the film may be obtained from equations 3 and 4. By the aid of these four equations, it is possible to determine in advance the important characteristics of any rotating mirror system that uses spherical lenses throughout.

It would seem to be desirable always to make the dimensions of the spot of light as small as possible and from the standpoint of geometrical optics there is no lower limit. However, because of the diffraction resulting from the limitation of the zone pencils at the edge of the mirror, it is impracticable to attempt to make the spot dimensions smaller than the first dark ring in the diffraction pattern. Thus, the size of the spot can never be less than is indicated approximately by

$$w_s \geqq \frac{2\lambda v}{w_3} \tag{8}$$

and

$$l_s \geqq \frac{2\lambda v}{l_3} \tag{9}$$

where λ is the effective wave length of the light used (approximately 0.00045 mm for ordinary photographic materials used with incandescent light sources). Because of the graininess and other characteristics of photographic materials, it is usually not advisable to attempt to make w_s or l_s smaller than 0.1 mm.

Now, in equation 6, it was assumed that the dimensions of the spot were somewhat larger than the limiting values given by equations 8 and 9. If the dimensions of the mirror w_3 and l_3 have already been fixed by mechanical or electrical considerations, we may then choose v, w_5 and l_5 quite freely, except for the limits set by equations 8 and 9 and the fact that w_5 and l_5 must ordinarily not be less than 0.1 mm. Since an increase in v decreases the illumination (equation 6) and at the same time increases the deflection (equation 7) it appears that a compromise between intense illumination and large deflection is necessary. In general, the desire for a large deflection is prompted by the demand for a high precision in reading the trace. However, the precision is not measured by the deflection alone but rather by the ratio of D to w_5, assuming that w_5 is greater than the photographic resolving power of the film. It is obvious that when v has such a value that w_5 is approximately 0.1 mm[5] any further increase in v will not increase the ratio D/w_5.

As a consequence, *there is an optimum value for v which should always be used unless special conditions make a different procedure advisable. Occasionally, it may be desirable to sacrifice precision for the sake of increased illumination but in most cases the value of v which corresponds to a minimum w_5 equal to* 0.1 *mm (equation 8) is the logical one to use. A larger v does not increase the precision and sacrifices illumination. A smaller v increases the illumination but at the expense of the precision.*

As an example of the foregoing, a well known make of oscillograph employs a mirror that is approximately 0.4 mm in width and 1.2 mm in length. By substitution in equation 8, we find that a spot 0.1 mm wide may be obtained when v is equal to or less than 44 mm. This is approximately one tenth of the distance from the mirror to the film in the commercial instruments as they are ordinarily supplied. These instruments are consequently wasteful of light and are needlessly expensive to operate because of the width of film required. It would seem that a more efficient apparatus might be constructed on a much smaller scale[6] which would then permit the trace to be recorded on motion picture film, using either negative or positive film depending upon the

[5] The average photographic material used for making negatives is capable of resolving at least 25 lines per millimeter under favorable conditions. The choice of 0.1 mm for the width of the line is consequently conservative.

[6] There is a saving in film not only because of the decrease in its width but because the speed of the film will be decreased proportionately also. Thus the area of film required is about proportional to the square of the deflection. These advantages usually more than compensate for the necessity of enlarging the records.

amount of light available. There is much to be gained by increasing α as equation 7 indicates but this is a mechanical rather than an optical problem.

CLASS II. THE OPTICAL SYSTEM CONSISTS OF CYLINDRICAL LENSES

Returning to the argument that follows equation 1, it will be recalled that the illumination was most intense when the source was focussed on the mirror and subsequently on the film but that this particular arrangement permitted no motion of the spot. It was also shown that all spherical lens systems, when compared on a basis of equal deflections, focussed the same density of flux on the surface of the photographic material. In other words, the mirror is compelled to be the aperture stop of a system of spherical lenses by the fact that we desire to record the amount of its rotation. Actually, this argument applies only to the dimension of the mirror normal to its axis of rotation. The other dimension of the mirror need not be the aperture stop of the system except for the fact that with spherical lenses there is no means of avoiding it.

It is here that cylindrical lenses are of distinct advantage as indicated in Fig. 3. The upper diagram shows a section in a plane through the mirror normal to the axis of rotation. This is the same general arrangement illustrated in Fig. 2. A diaphragm opening w_2 is focussed on the film by means of a cylindrical lens used as the window immediately

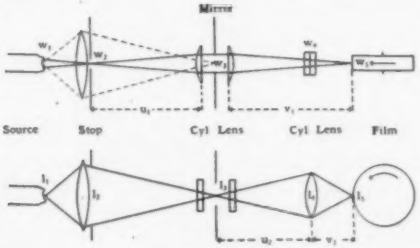

FIG. 3.

before the mirror. It is customary to use a spherical lens for this purpose because it is easier to mount but the cylindrical lens is equally efficient and is much simpler to consider theoretically (since the field and aperture stops in the two planes are entirely independent). As in the previous case, the focal length of the cylindrical lens may be obtained from

$$\frac{2}{f} = \frac{1}{u_1} + \frac{1}{v_1}$$

and the width of the spot on the film is

$$w_5 = w_2 v_1 / u_1 \tag{10}$$

In the plane through the axis of the mirror shown in the lower diagram of Fig. 3, the source of light is focussed on the mirror and the latter is in turn focussed on the film. The aperture stop in this plane is either l_2 or l_4, depending upon which stop or stop-image subtends the smaller angle at the film. It will be assumed in what follows that l_4 is the aperture stop and that l_2 and the lens preceding it are large enough to avoid obstructing the axial zone pencils. Obviously, the length of the spot on the film l_5 is given by

$$l_5 = l_3 v_2 / u_2 \tag{11}$$

(assuming that the image of the source more than fills the mirror).

The diaphragm width w_2 is assumed to be the source of light in the upper diagram of Fig. 3. Actually the source will in most cases be focussed on the mirror by means of a spherical lens as shown in Fig. 3. The brightness of the diaphragm is, of course, the same as the brightness of the source. However, care must be taken that the image of the source is larger than the mirror or the effective size of the mirror will be diminished. Except for this requirement the focal length of the spherical lens is not at all critical. The advantage of using this lens is seen from the lower diagram where it would generally be impossible to obtain a source with a length as great as l_2. When intense illumination is not required, the lens may be eliminated by using a ribbon filament incandescent lamp which can be placed in the position of the stop in Fig 3. In this case, l_2 represents the length of the filament and w_2 its width and the spherical lens is of course unnecessary.

The illumination on the film is given approximately by the product of the brightness of the source and the solid angle subtended by the cone of light which falls on the film.[7] Thus, with sufficient accuracy, the

[7] When the exit pupil is circular and of radius ρ and is situated a distance p from the film, the illumination on the film is

$$I = \pi B \left(\frac{\rho^2}{p^2 + \rho^2} \right)$$

For small values of ρ with respect to p this becomes

$$I = \pi B \frac{\rho^2}{p^2} = \omega B$$

where ω is the solid angle subtended at the film by the exit pupil. For small values of ω, this result is independent of the shape of the exit pupil. In the present case, the exit pupil in

expression for the illumination is (corresponding to equation (1) for the case of spherical lenses)

$$I = B \cdot \frac{l_4}{v_2} \cdot \frac{w_3}{v_1} \tag{12}$$

The deflection of the spot corresponding to a rotation α of the mirror is

$$D = 2 \cdot \alpha \cdot v_1 \tag{13}$$

CALCULATION OF EXPOSURE

It is interesting to compare systems of the first and second class by computing the exposure that may be obtained on the film under optimum conditions. Let s_1 represent the maximum transverse velocity of the spot normal to the direction of motion of the film. That is,

$$s_1 = \frac{dD}{dt} = 2 \cdot v \cdot \frac{d\alpha}{dt}$$

when the angular velocity of the mirror is a maximum. If E represents the exposure received by the film (lumen-seconds per cm^2) we have

$$E = I \cdot \frac{w_5}{s_1} = I \cdot \frac{l_5}{s_2} \tag{14}$$

wherein s_2 is the peripheral speed of the film. The usual custom is to make w_5 equal to l_5 (a square spot) and also to make s_2 equal s_1. If other conditions obtain, the second and third members of equations (14) will lead to different values of E and the lesser value should always be taken. By choosing s_2 equal to s_1, the maximum slope of the curve will be 45 degrees, which is ordinarily conducive to high precision in reading the trace.

The minimum exposure that a photographic material will record depends considerably upon the type of material employed. Let us designate the exposure that will leave a just perceptible deposit by σ, corresponding to the threshold exposure. The condition for sufficient exposure to make the trace readable is simply

$$E \geqq \sigma$$

one plane does not coincide with the exit pupil in the plane at right angles to it. The effective value of ω, however, is

$$\frac{L_4 \cdot w_3}{V_2 \cdot V_1}$$

as given by equation (12).

Substituting this in equation 14, we have as a necessary condition for successful records that

$$\frac{I \cdot w_5}{\sigma \cdot s_1} \geqq 1 \tag{15}$$

It is manifestly true from this that the conditions of maximum illumination will produce a maximum exposure. This has been tacitly assumed but it is now evident from an inspection of either equation 14 or 15.

Now, if the optical system is of the first class, the illumination (I) is obtainable from equation 6. Substituting, we have

$$\frac{B}{\sigma} \cdot \frac{w_3 l_3}{v^2} \cdot \frac{w_5}{s_1} \geqq 1 \tag{16}$$

as the condition of sufficient exposure to make the trace just readable. Values of B/σ are given in Table 1 for several combinations of different sources of light with different brands of photographic material. By substitution of the proper values of the quantities in equation 16, it is possible to ascertain in advance whether a particular optical system will yield satisfactory results. It should be noted, however, that the choice of w_5 is not free but is limited by the conditions of equation 8.

If the optical system is of the second class, the equation corresponding to equation 16 is

$$\frac{B}{\sigma} \cdot \frac{l_4}{v_2} \cdot \frac{w_3}{v_1} \cdot \frac{w_5}{s_1} \geqq 1 \tag{17}$$

The same limitation on the maximum value of w_5 applies in this case also but equation 9 is no longer necessary.

We may compare the efficiency of the two systems of illumination by dividing the left hand member of equation 17 by the left hand member of equation 16. If R is the ratio thus obtained, we have $R = (l_4/v_2) \div (l_3/v)$ after cancelling terms where possible and assuming the same spot deflection (and therefore that $v = v_1$). Now, the limiting value of l_4/v_2 is the aperture ratio of the cylindrical lens, which may very easily be as large as 1/4. If we assume the standard oscillograph mirror length to be three times its width and the optimum value of v to be used as already computed, R is then approximately 9 (assuming $l_4/v_2 = 1/4$). In other words, without going to any special pains to obtain a cylindrical lens with a larger aperture ratio than 1/4, it is

possible to obtain a satisfactory trace when the spot velocity is nine times the limit for a spherical lens system. When the conditions of the

TABLE 1.

This table gives approximate values of $B/\sigma \times 10^{-6}$. B is expressed in candles per cm² and σ in lumen-seconds per cm².	Sun	High intensity arc (*Crater*)	Ordinary arc (approx. 4000°K)	Tungsten (3200°K)	Tungsten (3000°K)	Tungsten (2800°K)
Super speed oscillograph film	72000	50000	16200	1570	800	400
Super speed motion picture film	72000	50000	16200	1570	800	400
Par speed motion picture film	36000	25000	8100	780	400	200
Par speed oscillograph film	24000	16700	5400	520	270	130
Kodak roll film	24000	16700	5400	520	270	130
Motion picture positive film	3600	2500	820	76	39	20
Bromide paper (average)	180	125	41	3.8	2.0	1.0
Developing out paper (average)	1.8	1.25	0.41	0.038	0.020	0.010

problem do not demand this additional speed, a spherical lens system is much simpler to arrange and avoids the slight distortion introduced by the cylindrical lens in front of the film.

SUMMARY

1. The optical system employed to record the deflection of a spot of light produced by a rotating mirror, has been studied. For convenience, two classes of systems have been identified. In the first, spherical lenses only are used. In the second, a combination of cylindrical lenses is employed. 2. It is shown that all properly designed spherical lens systems are equally efficient when compared on the basis of equal deflections. No improvement in illumination is possible over that obtained by using a single lens placed in any convenient position and arranged to focus an image of the source (or diaphragm near it) on the film. The diameter of this lens need be no larger than the greatest dimension of the mirror. 3. The above result is due to the fact that the mirror is the aperture stop of the system and must remain so because it is desired to measure the amount of its rotation. By the use of cylindrical lenses, the mirror may be made to remain the aperture stop in the required plane normal to the axis of suspension but need not remain the aperture stop in the plane of the axis. The illumination then depends on the aperture ratio of the cylindrical lens and, by making

it large, there is no difficulty in obtaining nine times as much light as with a spherical system. 4. The effect of diffraction at the edge of the mirror has been considered and is shown to result in an optimum value for the deflection. Attempting to increase the deflection beyond this limit does not increase the precision and diminishes the illumination. Decreasing the deflection yields more exposure but diminishes the precision. This result seems to have been quite generally overlooked as the present systems are very wasteful of light. 5. Equations have been derived for optical systems of both types which permit the size of the spot, its maximum permissible velocity, and its deflection for a given rotation of the mirror to be readily calculated. By the use of a table of photographic brightness values for different sources of light, it is possible to determine in advance whether a certain optical system will provide sufficient illumination to leave a readable trace under any set of conditions.

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OCTOBER 15, 1926

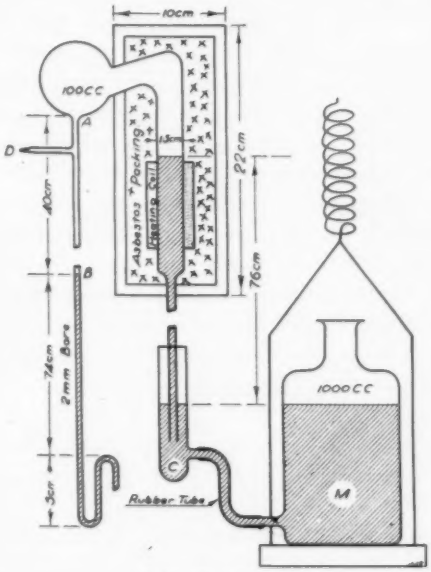
Photoelectric registering microphotometer.—The special feature of this apparatus is a resistance amplifier using a vacuum tube having two grids, thus making possible the use of low plate voltages. Amplification without distortion is possible, and with simple precautions the results are consistent and trustworthy. The interior grid and the plate are held at 3 volts and 13.5 volts respectively, by batteries. The photoelectric cell circuit contains the cell, a 130 volt battery, and a 50 to 200 megohm resistance. One end of this resistance is connected to the exterior grid, the other end through a biasing battery to the filament. Equilibrium is established in this circuit in 5 or 6 seconds. Values of amplified current through the galvanometer of ordinary type in the plate circuit are from 200 to 1000 times those in the photoelectric cell. Registration is effected by means of a beam of light reflected from the galvanometer mirror to a sensitive paper, the displacement of which is proportional to that of the plate being measured. The spectrum of Vega, a portion of the ultraviolet solar spectrum, and the rings in a Fabry and Perot etalon are shown as examples of performance.—[P. Lambert and C. Chalonge; Revue d'Optique *5*, pp. 404–420; 1926.]

G. W. MOFFITT.

A SIMPLE AUTOMATIC MERCURY STILL

By Howard L. Bronson

Mercury stills of various types and many designs are widely used in university and commercial research laboratories. In fact, there is scarcely a laboratory so small that it does not feel the need of some such device for purifying mercury, but most of those at present in use either require too much attention or withdraw too much mercury from circulation.



Automatic mercury still.

The accompanying figure illustrates a still that possesses certain very definite merits. (1) The mercury container M is supported by a spring of such length and stiffness that the mercury level in C is practically independent of the quantity of mercury in M. As far as the writer is aware, this is a new device which enables the still to operate continuously and without attention for as long as three or four days. If desired, the heating circuit can easily be arranged so that it will be opened automatically by the fall of the mercury level in C, in case the container M becomes empty. (2) The tube AB is made of such length

and size that it acts as a Sprengle pump and thus insures a permanent high vacuum above the mercury and consequently a low boiling point. (3) The volume of mercury permanently tied up in the still is only about 50 cc and this could readily be reduced to 20 cc by the use of still smaller apparatus. (4) It can be easily and quickly made in any laboratory at very small expense and the cost of operation is trifling.

The following details in addition to those furnished in the figure may be found helpful in constructing such a still: Pyrex glass was used throughout and the heating coil wound directly on it. The latter was made of about 45 feet of No. 26 oxidized "Ideal" wire having about 1.2 ohms per foot. The emf supply was taken from a small home-made transformer giving about 37 volts on the secondary. An ordinary "heavy duty" bell-ringing transformer would be quite suitable for the purpose. A fairly low voltage is desirable for much greater care would have to be taken with the insulation if the still was to be operated directly off a 110-volt lighting circuit. The power consumption in the above heating coil is about 25 watts and about 110 grams of mercury is distilled per hour. The still was originally exhausted through the side tube *D* by means of a common air pump.

The fact that the efficiency of the process is only about 40 per cent is a minor consideration when the total power consumed is so small. However, it could undoubtedly be considerably increased by taking sufficiently great care with the asbestos packing.

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[1] Since the above was written, a still has been made using a Ward Leonard Electric Co. 500-ohm. "Vitrohm" resistance for the heating unit. The porcelain tube was 4″ long and ¾″ internal diameter. It can safely be connected to the ordinary lighting circuit and still further simplifies the construction of the apparatus.

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